Uncertain Catastrophic Events: Another Source of Environmental Traps?

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Abstract

This paper aims to present another explanation for environmental inequalities, by the presence of catastrophic environmental events. We develop a simple dynamic model in which economy is subject to risk of catastrophic events. We show that implementing only adaptation policy could cause multiple equilibria, which we interpret it as an environmental inequality accross different regions or countries. Contrary to this result, it is shown that mitigation policy could save an economy from multiple equilibria. As a result, we show that adaptation and mitigation policy represents a trade-off concerning environmental inequalities. Based on this elements, we analyze the optimal policy mix of adaptation and mitigation activities, which could avoid these inequalities and show how optimally the policy mix can be implemented with taking into account the catastrophe probability. Our simulation results show that when economy faces a higher risk, both of policies increases but adaptation investment increases much more relatively to mitigation activity.

Keywords : Abrupt damage, Occurence Hazard, Multiple Equilibria, Inequality, Adaptation, Mitigation.

JEL Classification : D81,Q2, Q53, Q54,

1 Introduction

Why there exist environmental inequalities across different regions and countries in world ? Why some Sub-Saharan, Latin American and South Asian countries are suffering more from environmental catastrophes and low environmental conditions ? Are there any links between environmental conditions and income inequality [1] ? Many of explanations given for these questions by economists and researchers from other disciplines consists of high pollution emissions of developed countries [2] and the vulnerability of poor countries accompanied by the lack of giving efficient responses against the deterioration of environmental conditions. (see Stiglitz, 2014 ; Parks and Roberts, 2008)

Another possible explanation widely expressed consists of pollution haven hypothesis, which claims that manifacturers in developed countries seek to delocalize their production into low-cost labor countries. Consequently, pollution concentrates mostly in these countries where production costs are lower. (See Levinson and Taylor, 2008). As all of these explanations are based on supply-side of the economy, we think that is not giving an exact and entire explanation for existing environmental and consumption inequalities.

In this study, we focus on another aspect of occurence of environmental and income inequalities, in order to point out another possible source of environmental inequalities, which could be the catastrophic event risk. The mechanism behind is that countries where the resource stock is sufficiently conserved *(Note that*

[1] By income inequalities, we refer to consumption inequalities within this paper. In our model, income comes from the resource stock, from which environmental inequalities implies also consumption/income inequalities. Briefly, we can interpret consumption inequalities as income inequalities as both of them are highly correlated. See Aguiar and Bils (2011)

[2] Countries like China and India represent exceptional examples for this explanation but note that a substantial part of the pollution emitted by China comes from occidental multinational countries. See http://www.worldwatch.org/node/4764

we refer to environmental quality when we use the term resource stock), the hazard rate is sufficiently low since agents could be more productive and accumulate more human capital (See Zivin and Neidell, 2012) and could makes long term plans as they have a higher life expectancy. Therefore, agents in these regions are supposed to be more patient. Consequently, they will be more willing to postpone their consumotion, which case would allow a higher income/consumption level at the long run.

It is possible to consider the opposite mechanism. In countries where the environmental quality stock is less, the impatience level will be higher due to the higher hazard rate. Therefore, society will not postpone its consumption and exploit most of the resource at the begininning and have less to consume at the long run. Note that this explanation makes sense for Sub Saharan African countries where most of the income comes from exploitation of natural resources. In a world with higher frequency of catastrophic event risk, these countries will have a higher discount rate because of the low environmental conditions, which would probably trap them to lower life standards with low environmental quality and low consumption levels. As a result, there would be different and "polarized" income/environmental quality[3] equilbria across the world.[4] In order to understand the reasons behind multiple equibria, one can also say that there exists a trade-off not only between present and future consumption, but also between consumption and catastrophic event. This is one of the intuitive economic explanation for the existence of multiple equilibria.

Other important questions asked within this paper are ; "Can an appropriate environmental policy like adaptation and mitigation activities reduce or eliminate these inequalities ? Can these policies increase or decrease inequalities between different regions in world ? "How should a policy maker implement an optimal policy mix in order to eliminate these inequalities ?"

The subject of inequality and environmental policy has been treated in numereous empirical and theoretical studies. Constant (2015) analyzes the health inequalities by the means of multiple equilibria and shows an appropriate environmental tax policy could save the economy from a poverty trap. Torras and Boyce (1998) shows emprically that a more equal distribution of income tends to have a positive impact on environmental quality. McAusland (2003) shows in a theoretical model that income inequality can change the preferences for voting environmental policy and argues that low income groups could desire more strict environmental policy. Contrary to this study, Kempf and Rossignol (2005) argues that poorer the median voter, less she will devote resources for environmental policy, in order to increase its wealth. Boyce (2007) supports the idea that income and consumption inequalites are bad because of those who holds the capital stock will gain unequal gains from their economic activities with environmental degrading economic activities.

Our study differs in many senses from the cited literature about inequality and environment. We are focusing more on the link between pollution/income inequalities and catastrophic risk event, which can also explain the presence of inequalities explained by the mechanism analysed above.

There exists a substantial literature in climate change economics on the links between catastrophic events and optimal environmental policy design. Especially, the subject has been treated in a way that analyzes the costs of earlier policy interventions with the cost of a later intervention. (See Stern, 2007 ; Nordhaus,2008). However, these two prominent studies and many other studies on this branch of the literature does not take into account the effects of uncertainty on the design of an optimal environmental adaptation and mitigation policy. Useful definitions for these policies are made in the following way ; adaptation policy aims at reducing vulnerability of society against to negative impacts of climate change while mitigation decreases the risk of an abrupt catastrophic event but not eliminate it completly. Bréchet et al. (2012) , Ayong Le Kama and Pommeret (2014), Kane and Shogren (2014) and Buob and Stephan (2010) are first analytical studies that treats optimal design of an environmental policy on adaptation and mitigation. This line of the literature assumes implicitly that climate change is not an abrupt incident but a gradual one and damages are reversible.

Neverthless, some more recent literature adopts a different point of view, with defending the idea that catastrophic events are abrupt events and may cause irreversible damages. (See Clarke and Reed, 1994;

[3]The term environmental quality and resource stock are used interchangely in this article.

[4]Note that this mechanism is possible if only there exists an endogeneous catastrophic event probability, in which case agents in society would face a trade-off between consumption and catastrophic event. In the contrary case, the economy will have a unique equilibrium.

Bommier et al, 2015; Gjerde et al., 1999; Tsur and Zemel, 1996, 1998, 2008,2009,2015 ; Zemel, 2015 and the references there in.) At that point, Weitzmann (2009) insists on fat tail risk events that occurs rarely but causes huge environmental and economic damages and argues that usual models policy implications could not be adequate for controling low probability high impact environmental events. Therefore, there exists not only a intertemporal trade-off between present and future consumption but also a trade-off between today's consumption and catastrophic event.

We observe that none of these studies focus on an analysis of multiple equilibria in an economy facing catastrophic event risks that represent inequalities across different regions of countries. To the best of our knowledge, only study working on multipe equilibria without any aspect of risk on a different framework than ours and making a deterministic analysis is Schumacher (2009). The author argues that twin-peaks of economic growth can be also explained by endogeneous discounting on capital and the solution to have a convergence of wealth could be achieved through an exogeneous improvement in technology.

The contribution of this study is twofold ; Firstly, we show on a simple non-deterministic dynamic model that environmental and income inequalities could be explained by the risk of an abrupt catastrophic. We show that the possibility of having inequalities disappears if there is not an endogeneous catastrophic event probability. Economically, this fact can be explained by the absence of the trade-off between catastrophic event and consumption when there is not an endogeneous catastrophic event risk.

Secondly, we show that adaptation and mitigation policies have opposite effects on environmental and income inequalities, which represent a different trade-off that is not analysed in literature to the best of our knowledge.

In our framework, we argue that a social planner implementing only adaptation policy could cause income and environmental inequalities. This fact is due to the fact that adaptation policy decreases the resource stock (or environmental quality) as we show in our model. We show that an economy adapting to climate change increases its pollution level as it assumes a higher quantity of penalty after occurence of a catastrophic risk. Therefore, the catastrophic risk increases and causes environmental inequalities across different regions and countries.

Contrary to this effect of adaptation policy, we argue that mitigation policy could save an economy suffering from multiple equilibria. This case is possible as mitigation aims at reducing hazard rate by increasing the environmental quality, which give an opportunity to developping countries to be more productive and accumulate human capital. The position we take for these policies does not mean that we reject adaptation policy. On the contrary, adaptation policy aiming at reducing damages from a potential catastrophic is crucial because it is not possible to eliminate totally the risk of a catastrophic event even with mitigation activity.

When we look at IPCC report - Climate Change 2014: Impacts, Adaptation, and Vulnerability (2014) and at decisions taken recently at COP21 held in Paris, there is much more accent on adaptation activities and objectives on mitigation activities are not clear.

2014 IPCC report is highlighting the fact that society should invest heavily on adaptation activities The report claims that even if develeoped countries take the necessary measures in order to cut their emissions, the positive effects of this action will appear after some decades. Another strong claim of this report is that developping countries are mostly vulnerable to climate change more for other reasons than greenhouse gas emissions.

Basing on an increased accent on adaptation policy and on the arguments that we present, we argue that a policy maker must be cautious when she implements adaptation and mitigation policies because an excessive adaptation investment could trap an economy to multiple equilibria, as we show in this current study. We support the idea that adaptation policy must be coupled by an adequate mitigation policy, as these two policies are shown to have opposite effects on inequalities.

We also conduct a numerical exercice on our model. We show that economy increases its level of adaptation and mitigation activity when it faces a higher risk of catastrophe. However, the adaptation investment increases much more relatively to mitigation activity. Thus, a policy maker must be cautious when she implements this two policies in an economy with higher risks. An increasing risk will imply more adaptation capital, which increases the catastrophic event rate by decreasing the resource stock, and would require afterwards much more adaptation capital. As a result, the catastrophic event probability by itself, yielding a such vicious cycle of more "adaptation capital-less resource stock" effect, would cause inequalities and also increase the frequence of catastrophic events. Another result from numerical exercice shows that in an economy suffering from multiple equilibria due to a high adaptation activitiy, the regions trapped at low equilibrium accumulates much more adaptation capital compared to countries with high level of resource stock and consumption. For this reason, the proposition of IPCC and decisions taken in COP21 must be analyzed more carefully. Advocating more adaptation activity without coupling it by mitigation activities (or without advocating mitigation efforts to developped countries) to developping countries could trap this fragile regions to low steady state of environmental quality and consumption. As a result, implementing only adaptation policy without insufficient mitigation efforts to challenge climate change can be another source of inequalities and climate change concerns.

The remainder of the paper is organized as follows. The following section presents the benchmark model with analyzing the sufficiency conditions for having a multiple equilibria economy. Section 3 explains the model with adaptation policy and derives the analytical condition how this policy could cause a multiple equilibria economy, with also analyzing its effects on phase diagram on the plane (c, S). Section 4 presents the model with mitigation policy and shows the condition how this policy could save an economy from multiple equilibria and makes also a phase diagram analysis. Section 5 analyzses the model with both adaptation and mitigation policies and derives the multiple equilibria condition and shows that having a unique or multiple equilibria depends on which policies effect dominate the other one. Section 6 presents the numerical illustration with the calibration and functional specifications and justifies the theoretical findings of previous section. Last section concludes the paper.

2 Model

We propose a simple growth model with endogeneous catastrophe probability similar to Tsur and Zemel (2006) ;

$$\underset{c_t}{max} E_T \left[\int_{\tau}^{T} u(c_t) e^{-\rho(\tau-t)} dt + \varphi(S_T) e^{-\rho(T-t)} \right] = \int_{t}^{\infty} \left[u(c_\tau) \frac{(1-F(\tau))}{(1-F(t))} + \frac{f(\tau)}{(1-F(t))} \varphi(S_\tau) \right] e^{-\rho(\tau-t)} d\tau \tag{1}$$

where $h(S_t) = \frac{f(t)}{1-F(t)} = -\frac{d[ln(1-F(t))]}{dt}$ which gives ;

$$F(t) = 1 - e^{-\Omega(t)} \ \ and \ \ f(t) = h(S_t) e^{-\Omega(t)} \tag{2}$$

We introduce the post value function in model in line with with numerous paper as Bommier et al. (2015), Tsur and Zemel (2015), Tsur and Zemel (1996), Tsur and Zemel (2006), Clarke and Reed (1994) and Cropper (1976). After the occurence of catastrophe, society is exposed to some constant-level penalty ψ and consumption level is reduced to some c_{min} which represents consumption that is essential for survival but this level of consumption does not provide any utility. This corresponds to a "doomsday" event in which the value of the problem after catastrophe reduces to zero, as also stated in Tsur and Zemel (2006).

$$\varphi(S) = \int_{0}^{\infty} u(c_{min}) e^{-\rho t} dt - \psi = -\psi \tag{3}$$

where $u(c_{min}) = -\psi$.[5] By integrating by parts the equation (1), this term reduces to the following expression with endogenous discount which encompasses the endogeneous catastrophic event probability ;

$$\underset{c_t}{max} \int_{t}^{\infty} \left[u(c_\tau) + h(S_\tau) \varphi(S_\tau) \right] e^{-z(t,\tau)} dt = \int_{t}^{\infty} \left[u(c_\tau) - \psi h(S_\tau) \right] e^{-z(t,\tau)} dt \tag{3}$$

[5]This feature does not cause any lose of generality of our results.

$$z(t,\tau)=\rho(t-\tau)+\Omega(t)-\Omega(\tau)=\int_t^\tau \rho+h(S_x)\,dx=\int_t^\tau \theta(S_x)\,dx \tag{4}$$

where we define $\theta(S)=\rho+h(S)$ in order to keep the notation simpler for the remainder of the paper. The environmental quality (or renewable resource) S_t evolves in the following way ;

$$\dot{S}_t=R(S_t)-c_t \tag{5}$$

By integrating by parts the equation (1), this term reduces to the following expression ;

$$\underset{c_t}{max}\int_t^\infty (u(c_\tau)-\psi h(S_\tau))\,e^{-z(t,\tau)}d\tau \tag{6}$$

$$z(t,\tau)=\rho(t-\tau)+\Omega(t)-\Omega(\tau)=\int_t^\tau \rho+h(S_x)\,dx=\int_t^\tau \theta(S_x)\,dx \tag{7}$$

where we define $\Omega(t)=\int_0^t h(S_\tau)\,d\tau$ and $\theta(S)=\rho+h(S)$ in order to keep the notation simpler for the remainder of this paper, where ρ stands for the pure rate of time preference.

Assumption 1 :
(i) We assume a twice continously differentiable utility function with following properties ; $u^{'}(c)>0$, $u^{''}(c)<0$ and $\underset{c\to 0}{lim} u^{'}(c)=\infty$.
(ii) The endogenous hazard rate $h(S)$ is twice continously differentiable with following properties ; $h(S)>0$, $h^{'}(S)<0$ which implies the same properties directly for the discount rate $\theta(S)$ defined above.
(iii) $R(S)>0$, $R^{'}(S)>0$ and $R^{''}(S)<0$

In order to maintain the optimization program simpler, we transform the problem by Uzawa's virtual time method. This transformation would be useful to make a phase diagram anaylsis, keeping a two dimensional dynamic system in benchmark model. (Note that same calculations could be obtained exactly in the same manner with using dynammic programming as in Schumacher (2009).)

$$\underset{c}{max}\int_t^\infty \left[\frac{u(c)-\psi h(S)}{\theta(S)}\right]e^{-q}d\theta \tag{9}$$

subject to the budget constraint ;

$$\frac{dS}{dq}=\frac{R(S)-c}{\theta(S)} \tag{10}$$

Recall that $\frac{dq}{\theta(S)}=dt$ which will be useful after in order to convert the dynamic of the transformed shadow price into the classical one. The Hamiltonian of the problem is the following ;

$$\mathcal{H}=\frac{u(c)-\psi h(S)}{\theta(S)}+\lambda\left[\frac{R(S)-c}{\theta(S)}\right] \tag{11}$$

The optimal solution is as following ;

$$\frac{\partial\mathcal{H}}{\partial c}=\frac{u^{'}(c)}{\theta(S)}-\frac{\lambda}{\theta(S)}=0 \tag{12}$$

$$\frac{\partial\mathcal{H}}{\partial S}=\lambda-\frac{d\lambda}{dq}=-\left(\frac{\theta^{'}(S)}{\theta^2(S)}u(c)-\psi h(S)\right)-\frac{\psi h^{'}(S)}{\theta(S)}+\frac{\lambda R^{'}(S)}{\theta^2(S)}-\lambda\frac{\theta^{'}(S)}{\theta^2(S)}[R(S)-c] \tag{13}$$

By multiplying this term by $\theta(S)$, we convert the problem from virtual time to the real time ;

$$\theta(S)\lambda-\frac{d\lambda}{dt}=-\frac{\theta^{'}(S)}{\theta(S)}(u(c)-\psi h(S))-\psi h^{'}(S)+\lambda R^{'}(S)-\frac{\lambda\theta(S)}{\theta'(S)}[R(S)-C] \tag{14}$$

From which we can write down the optimal consumption dynamics and the dynamics of environmental quality. This two dimensional dynamic system describes the economy and also similar to Schumacher (2009) and Strulik (2012) ;

$$\dot{c} = -\frac{u^{'}(c)}{u^{''}(c)}\left[R^{'}(S) - \theta(S) - \frac{\psi h^{'}(S)}{u^{'}(c)} - \frac{\theta^{'}(S)}{\theta(S)}\left[\frac{u(c) - \psi h(S)}{u^{'}(c)} + \dot{S}\right]\right] \tag{15}$$

$$\dot{S} = R(S) - c \tag{16}$$

Note that the same dynamics can also be obtained by the transversality condition $\underset{t\to\infty}{lim}\mathcal{H} = 0$ (See Michel (1982) and Ayong Le Kama and Schubert (2008)). At the steady state $\dot{c} = \dot{S} = 0$, we have the following values ;

$$R^{'}(S) - \theta(S) - \frac{\psi h^{'}(S)}{u^{'}(c)} - \frac{\theta^{'}(S)}{\theta(S)}\left[\frac{u(c) - \psi h(S)}{u^{'}(c)} + \dot{S}\right] \tag{17}$$

$$R(S) - c = 0 \tag{18}$$

Proposition 1. *Multiple steady state (i.e environmental trap) possibility arises when there exists an endogeneous probability of catastrophic event risk. The existence of multiple steady state is not possible if there does not exist any endogeneous catastrophic risk. Equivalently, lower risk means that there are less chances to have multiple equilibria.*

Proof. (a) The proof starts by analyzing the limits of function $G(S)$ and the second part of the proof focuses on the form of function $G(S)$ and necessary conditions for multiple steady state, which is crucial to draw the phase diagram of the dynamic system.

We could describe the steady state of this economy by a single equation as a function of S. Let's say this function $G(S)$ which is (by replacing c by $R(S)$) Before starting to analyze the necessary conditions for the existence of multiple steady state, we must analyze how this function behaves when $S \to 0$ and $S \to \bar{S}$. Note that $\bar{S}$ is the positive environmental quality level at which $R\left(\bar{S}\right) = 0$.

In some sense, the function $G(S)$ could be considered as the equation $\dot{c} = 0$ as a function of S ;

$$G(S) = R^{'}(S) - \theta(S) - \frac{\psi h^{'}(S)}{u^{'}(R(S))} - \frac{\theta^{'}(S)}{\theta(S)}\left[\frac{u(R(S)) - \psi h(S)}{u^{'}(R(S))}\right] \tag{19}$$

In this case, it is easy to see that[6]

$$\underset{S\to 0}{lim} G(S) = m > 0$$

$$\underset{S\to \bar{S}}{lim} G(S) = z < 0$$

Note that we limit our analysis between $S \in \left[0, \bar{S}\right]$. We can easily say that the function $G(S)$ starts with a positive value and tends to be negative when S approaches $\bar{S}$.

(b) In this part, we show the necessary conditions for the existence of multiple steady state, which allows us also to represent the function $G(S)$ on a plane $(G(S), S)$;

[6]More concretely, it is easy to see the limits with given functions for $h(S)$ and $R(S)$ as in Ren and Polasky (2014), $h(S) = \frac{2\bar{h}}{1+exp\left[\eta\left(\frac{S}{S^*}-1\right)\right]}$ and $R(S) = S + gS\left(1 - \frac{S}{\overline{S}}\right)$ where $\overline{S}$, η and g are the carrying capacity, risk endogeneity and intrinsic growth rate respectively.

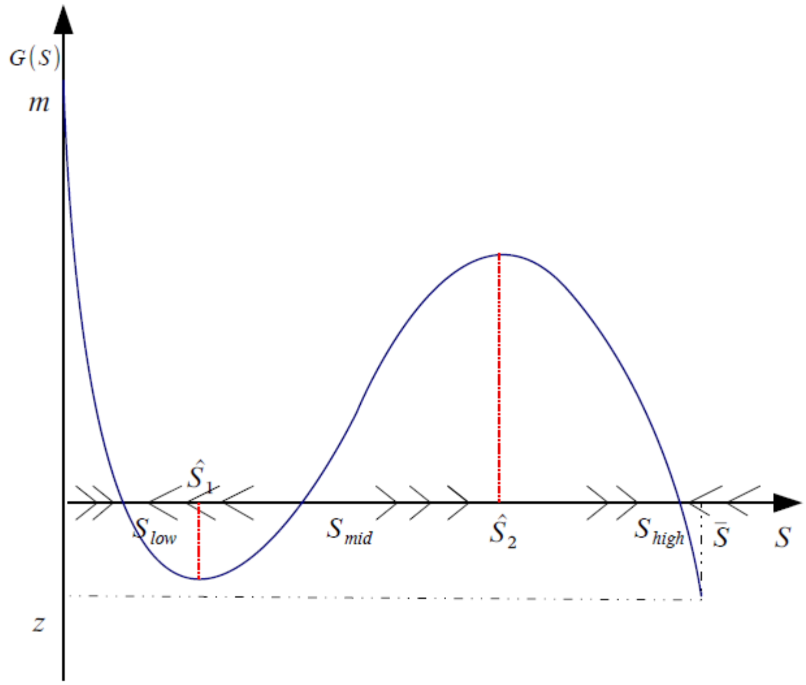


Figure 1 : $G(S)$ function with uncertainty

The sufficient condition for the existence of S_{low} is that $\exists S < \widetilde{S}$ (this $\widetilde{S}$ will be between S_{mid} and S_{high}) which ensures (i) $G^{'}(S) > 0$ and (ii) $G(S) < 0$. When we look at the graphic, this condition makes sense. If we don't have (i) $G^{'}(S) > 0$, the function $G(S)$, starting from m will cross x-axis just one more time and converges to z, which will make a unique steady state. Also the condition (ii) $G(S) < 0$ is necessary for that the function $G(S)$ cross the x-axis by S_{low} at least one time.

Recall that points $\hat{S}_1$ and $\hat{S}_2$ are points of inflection. After these points, first derivative of function $G(S)$ changes sign. Taking into account these points in our analysis is important in order to precise the directions of arrows on the phase diagram.

The necessary condition for the existence of S_{high} is that $G\left(\widetilde{S}\right) > 0$. This one also makes sense. If this condition does not hold, we could have a function $G(S)$ not crossing x-axis for the second time which converges directly to z without changing sign once it becomes negative after being positive. Once we have the condition $G\left(\widetilde{S}\right) > 0$, we will see that the function $G(S)$will for sure cross x-axis by S_{mid} after will tend to z when S approaches $\bar{S}$.

With these necessary conditions, it is possible to say that there will be three steady states, one being unstable and two others being stable.

It is also possible to analyze on the graphic above the stability of the different equilibrium. The middle steady-state (for S_{mid}) is not stable. As the function $G(S)$ is increasing, we will always go far from S_{mid} as the arrows shows on the graphic. If we have $S > S_{mid}$, we converge to S_{high}. Otherwise, we converge to S_{low}.

The sufficient condition (i) is checked by

$$G^{'}(S) = R^{''}(S) - \theta^{'}(S) - \left[\frac{\theta^{''}(S)\,\theta(S) - \left(\theta^{'}(S)\right)^2}{\left(\theta(S)\right)^2}\right] \frac{u(c) - \psi h(S)}{u^{'}(c)}$$

$$-\left[\frac{\theta^{'}(S)}{\theta(S)}\left(1-\frac{(u(c)-\psi h(S))\,u^{''}(c)}{\left(u^{'}(c)\right)^{2}}\right)+\frac{\psi u^{''}(R(S))\,h^{'}(S)}{\left(u^{'}(c)\right)^{2}}\right]\left[R^{'}(S)\right]>0 \tag{20}$$

It is obvious that when there does not exist an endogeneous probability of catastrophe, all the terms with hazard rate in the LHS (left hand side) vanish and we have $R^{''}(S)+\frac{\chi u^{''}(R(S))}{\left(u^{'}(R(S))\right)^{2}}>0$, which is impossible. In this case, the model reduces completely to a standard simple growth model without a possibility of multiple equilibria. This completes the proof.

The $G(S)$ function without any risk is a decreasing function ;

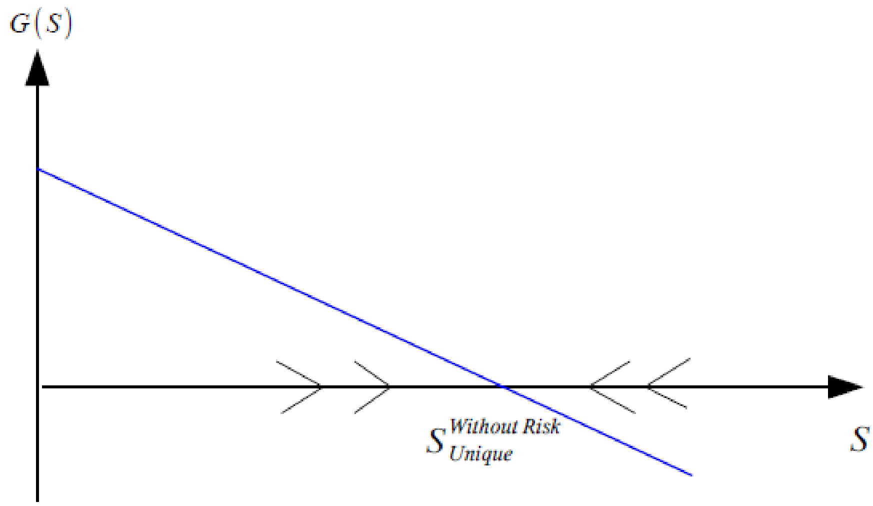


Figure 2 : $G(S)$ function without uncertainty.

The economic intuition behind the environmental inequalities comes from the fact that in some countries where the marginal utility of consumption is sufficiently high [7], agents are highly impatient and they direct their consumption toward today and consequently consume less in future, which case is also stated in a different framework in Schumacher (2009). This implies that for survival reasons, poor countries will consume a substantial part of their resources at the beginning and will have less to consume at the long run. One can also explain why some countries trap to the "bad" equilibrium at which the environmental quality and consumption levels are low at the steady state. As poor countries, consume a bigger part of the resources due to the reasons aforementioned and to low initial resource stocks, they find themselves very fastly in lower environmental conditions, which case also increases their discount rate [8] due to increasing hazard rate, which could push again these poor countries to consume much more today rather than tomorrow, in which case the environmental quality will again detoriarate much more. This yields a vicious cycle from which some countries could not escape. But we will see that this kind of amplified effect counts much more for present for countries with lower marginal utility where natural capital is sufficiently high.

In regions like Sub-Saharan Africa, South Asia and Latin America, the most part of national income comes from the exploitation of natural resources (like forests and mines). Due to high catastrophe probability, these countries will exploit natural resrouces more extensively, by increasing again the catastrophe probability. This ones explains why most of catastrophes takes in these places. Margono et al. (2014) show that deforestation was even much more in Indonesia than in Brasil between the period 2000-2012 and there was a loss of %40 of total national forests. In fact, the higher impatience level of these countries implies that these countries,

[7] i.e consumption level is close to subsistence level

[8] Note that the discount rate mentionned here is composed by a pure rate of time preference augmented by the hazard rate.

in their intertemporal decisions, are less sensitive to variations of hazard rate. [9] In order to deepen our understanding about this fact, we rewrite the Euler equation in a similar way as made in Schumacher (2009) ;

$$\dot{c} = -\frac{u^{'}(c_t)}{u^{''}(c_t)}\left[R^{'}(S_t) - \theta(S_t) - \frac{\psi h^{'}(S_t)}{u^{'}(c_t)} - \frac{\theta^{'}(S_t)}{u^{'}(c_t)}\int_t^{\infty}\left(u(c_\tau) - \psi h(S_\tau)\right)e^{-\Omega_\tau}d\tau\right] \tag{21}$$

It is straightforward to see from the last expression in this reformulated Euler equation (21) that when marginal utility of consumption is sufficiently high, the variations in hazard rate will be less important and agents will be less sensitive to changes in environmental quality. Alternatively, this means that people in poor countries don't take into account the hazard rate in their interemporal decisions as much as countries with higher life standards take into account. However, this does not mean that hazard rate does not play any role concerning the occurence of inequalities of environmental quality accross different regions of the world. Only in case of sufficiently high hazard rate, the vicious cycle effect will be present and trap poor countries to "bad" steady state. Otherwise, the economy will have a unique equilibrium without endogeneous risk.

While poor countries will consume most of their resources at the begining for survival, they will consume less at the steady state with a lower environmental quality. The consumption will start relatively from a higher level and will decrease or increase very slightly through the steady state.

In today's world, the forests cover % 30.88 [10] of the world's land surface and this rate decreases each year according to data collected by Food and Agriculture Organization of the United Nations (FAO). Consequently, hazard rate for the occurence of catastrophic events increases, which indeed increases the possibilities to have environmental inequalities across different regions, with a higher frequence of environmental catastrophes in some regions. [11]

In some sense, we argue that catastrophic event probability itself increases the frequency of environmental catastrophes by the channel of trapping countries in Sub Saharan Africa, Latin America and in South Asia to low environmental quality, which implies directly a higher frequency level of environmental catastrophes.

2.1 Phase Diagram Analysis

In this section, we show how we define rigourously the arrows on the phase diagram. If (c, S) is below the $\dot{S} = 0$, $R(S) - c > 0$, which makes $\dot{S} > 0$ and vice versa. The analysis is not that easy for $\dot{c} = 0$ but we could find the direction of arrows with using the multiple steady state condition. Above the curve, we will have $\dot{c} > (<)\, 0$ if $G^{'}(S) > (<)\, 0$. We can see that $\dot{c} > 0$ for the middle steady state. We precise different zones on the phase diagram, for which zones, the first derivative of function $G(S)$ changes.

The shape of the curve $\dot{c} = 0$ is also important to be analyzed. Its shape depends on how this curve behaves (increase or decrease) according to environmental quality $\dot{S}$. In order to see this one, we can use implicit function but we must be cautious because the curve $\dot{c} = 0$ could have a non-monotone behavior. In this section, we analyze in which regions, this curve is increasing or decreasing. For this one, let's write the implicit function, in order to see the slope of the curve $\dot{c} = 0$ according to environmental quality S.

$$\left.\frac{dc}{dS}\right|_{\dot{c}=0} = -\frac{R^{''}(S) - \theta^{'}(S) - \left[\frac{\theta^{''}(S)\theta(S) - \left(\theta^{'}(S)\right)^2}{(\theta(S))^2}\right]\frac{u(R(S)) - \psi h(S)}{u^{'}(R(S))} - \frac{\psi h^{''}(S)}{u^{'}(R(S))}}{-\frac{\theta'(S)}{\theta(S)}\left(1 - \frac{(u(c) - \psi h(S))u''(c)}{\left(u'(c)\right)^2}\right) + \frac{u''(c)h'(S)}{\left(u'(c)\right)^2}} \gtrless 0 \tag{22}$$

[9]Equivalently, these countries are less sensitive to the environmental quality as they try to fulfill their essential needs for survival.

[10]See http://data.worldbank.org/indicator/AG.LND.FRST.ZS

[11]Just in 2010, according to data from OECD, around 2 millions of people have died due to environmental reasons (mainly air pollution) in India and China.

It is easy to see that the denominator is always negative. Note that, from multiple steady state condition, between $\left[0;\hat{S}_1\right]$ and $\left[\hat{S}_2;\bar{S}\right]$, we know that $G^{'}(S)<0$. Between these two intervals, we can see immediately that the slope of the curve $\dot{c}=0$ is always positive, which means that the curve is always increasing between this interval. Conversely, between the intervals $\left[\hat{S}_1;\hat{S}_2\right]$, we don't know exactly the sign of the nominator of $\frac{dc}{dS}\big|_{\dot{c}=0}$ which means that it is either increasing or decreasing. In this case, we would have an increasing or decreasing for $\dot{c}=0$ between $\left[\hat{S}_1;\hat{S}_2\right]$. As a nutshell, we will have two different configurations for the phase digram. The first diagram is with a monotonic increasing curve $\dot{c}=0$ curve for $\left[\hat{S}_1;\hat{S}_2\right]$.

The phase diagram of the dynamical system is the following one ;

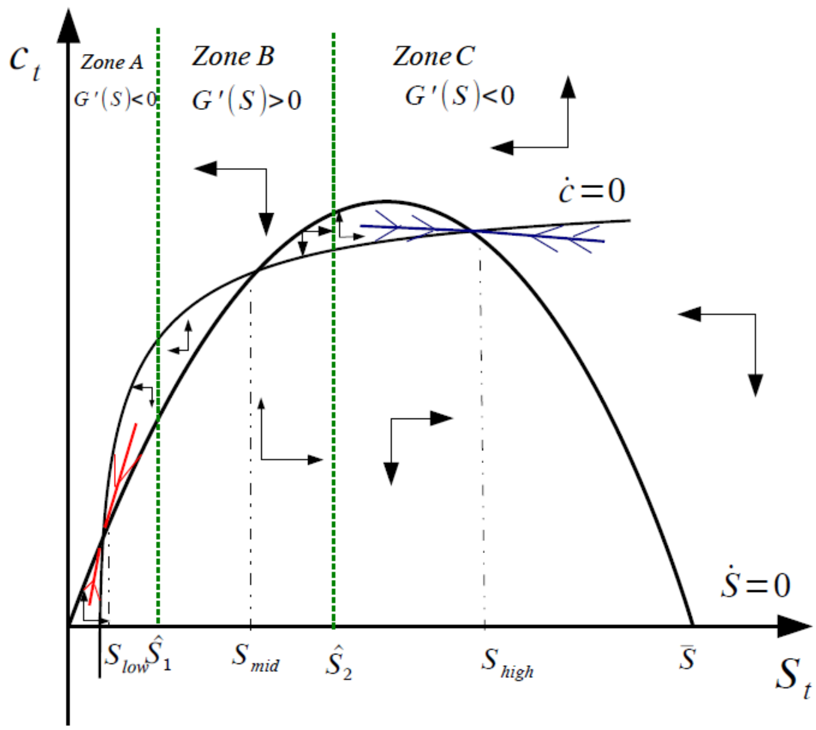


Figure 3 : Phase diagram with monotonically increasing $\dot{c}(S)=0$ curve

- Zone A $\in \left[0;\hat{S}_1\right]$
- Zone B $\in \left[\hat{S}_1;\hat{S}_2\right]$
- Zone C $\in \left[\hat{S}_2;\bar{S}\right]$

It is easy to see this at the zone B, the function $G(S)$ is increasing (i.e $G^{'}(S)>0$) between $\hat{S}_1$ and $\hat{S}_2$. One may see this also on the graphic of $G(S)$ function. When we look at zone A and zone C , we will have $\dot{c}<0$ (i.e $G^{'}(S)<0$) for the low and high steady state. With all these informations, it is straightforward to draw the arrows on the phase diagram.

The second phase diagram is as follows ;

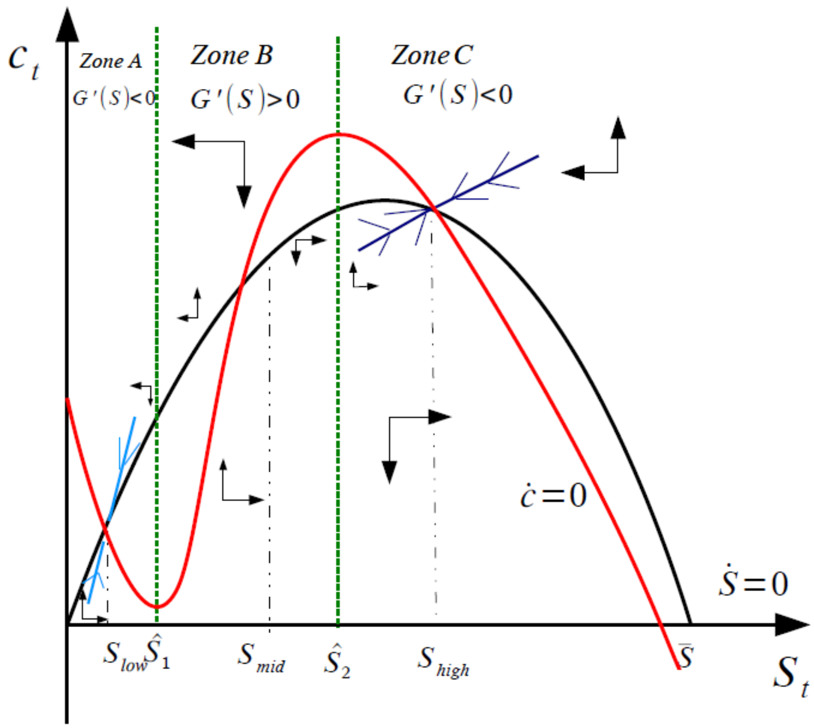


Figure 4 : Phase diagram with non-monotonically increasing $\dot{c}(S) = 0$ curve

The fact that $\dot{c}(S) = 0$ curve is non-monotonically decreasing does not change the transitional dynamics of the model but for sure, the middle steady state could change. The differential system describing the economy can be written as follows ;

$$\begin{bmatrix} \dot{c} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} \frac{d\dot{c}}{dc} & \frac{d\dot{c}}{dS} \\ \frac{d\dot{S}}{dc} & \frac{d\dot{S}}{dS} \end{bmatrix}_{\dot{c}=0,\dot{S}=0} \begin{bmatrix} c - c^* \\ S - S^* \end{bmatrix}$$

$$\frac{d\dot{c}}{dc} = \theta(S) - R'(S)$$

$$-\frac{u'(c)}{u''(c)}\left[R''(S) - \theta'(S) - \frac{\psi h''(S)}{u'(c)} - \left(\frac{\theta'(S)}{\theta(S)} - \frac{\left(\theta'(S)\right)^2}{\left(\theta(S)\right)^2}\right)\left[\frac{u(c) - \psi h(S)}{u'(c)}\right] - \frac{\theta'(S)}{\theta(S)}R'(S)\right]$$

$$\frac{d\dot{S}}{dc} = -1$$

$$\frac{d\dot{S}}{dS} = R'(S)$$

We know that for a saddle-stable path system, it is necessary to have one positive and one negative eigenvalue, denoted $\mu_{1,2}$. As the $Tr(J) = \mu_1 + \mu_2$ and $Det(J) = \mu_1\mu_2$. It is sufficient to show that $Tr(J) > 0$ and $Det(J) < 0$. It is easy to see that $Tr(J) = \theta(S) > 0$ and with arranging the terms for the determinant, we can see that determinant reduces to the multiple steady state condition $G(S)$. We conclude that $Det(J)$ is negative if

$$G^{'}(S) = R^{''}(S) - \theta^{'}(S) - \left[\frac{\theta^{''}(S)\,\theta(S) - \left(\theta^{'}(S)\right)^2}{(\theta(S))^2}\right]\frac{u(R(S)) - \psi h(S)}{u^{'}(R(S))} - \frac{\psi h^{''}(S)}{u^{'}(R(S))} + \frac{\psi u^{''}(R(S))}{(u^{'}(R(S)))^2}$$

$$- \left[\frac{\theta^{'}(S)}{\theta(S)}\left(1 - \frac{(u(c) - \psi h(S))\,u^{''}(c)}{(u^{'}(c))^2}\right)\right]\left[R^{'}(S)\right] < 0 \tag{23}$$

This condition holds definitely for low and high steady states with the assumption made on the level of post-value function above, which means that there is no possibility of complex dynamics for these steady states. Complex dynamics arises if $(Tr(J))^2 - 4Det(J) < 0$. As $Det(J)$ is negative for low and high steady states, this prevents these to steady states to have complex dynamics. However, for the middle steady state there is a possibility to have complex dynamics arises if

$$(\theta(S))^2 < 4\frac{u^{'}(c)}{u^{''}(c)}\left[R^{''}(S) - \theta^{'}(S) - \frac{\psi h^{''}(S)}{u^{'}(c)} - \left(\frac{\theta^{'}(S)}{\theta(S)} - \frac{\left(\theta^{'}(S)\right)^2}{(\theta(S))^2}\right)\left[\frac{u(c) - \psi h(S)}{u^{'}(c)}\right] - \frac{\theta^{'}(S)}{\theta(S)}R^{'}(S)\right] \tag{24}$$

3 Model with Adaptation Policy

In this section, we analyze the benchmark model with adaptation capital K_A and show how qualititative results could change according to the original problem and in which cases this policy could trap a country to a low consumption and low environmental quality steady state. The social planner seeks to solve the following maximization problem, similar to the first problem ;

$$\underset{c_t}{max} E_t\left[\int_{\tau}^{T}(u(c_t) - Q(A_t))\,e^{-\rho(\tau-t)}dt + \varphi(S_T, K_{A_T})\,e^{-\rho(T-t)}\right]$$

$$= \int_{t}^{\infty}\left[(u(c_\tau) - H(A_\tau))\frac{(1-F(\tau))}{(1-F(t))} + \frac{f(\tau)}{(1-F(t))}\varphi(S_\tau, K_{A_\tau})\right]e^{-\rho(\tau-t)}d\tau \tag{25}$$

where $H(A)$ is the convex cost function for adaptation investment as a function of adaptation investment A. In this model with adaptation capital K_A, the form of the post value function changes and takes the form ;

$$\varphi(S, K_A) = \int_{0}^{\infty} u(c_{min})\,e^{-\rho t}dt - \psi(K_A) = -\psi(K_A) \tag{26}$$

As shown above in the post-value function, the penalty inflicted to society is not anymore a constant parameter but a function of adaptation capital as in Zemel (2015). This means that the scale of the penalty is a function of adaptation capital K_A. It is worthwile to discuss this specification of adaptation policy on penalty. The benefits of adaptation capital are present just after the occurence of catastrophe. Thus, adaptation capital has no prior direct positive effect on economy as in Zemel (2015). Alternatively, we are considering adaptation policy as a proactive investment. (See Zemel, 2015 ; Smith et al., 2000; Shalizi and Lecocq, 2000). We can remark that with this specification, the effect of hazard rate enters directly in optimal dynamics in the model. Integrating by parts the social planner's program, we have

$$\underset{c_t}{max}\int_{t}^{\infty}[u(c_\tau) + h(S_\tau)\,\varphi(S_\tau, K_{A_\tau}) - Q(A_\tau)]\,e^{-z(t,\tau)}dt = \int_{t}^{\infty}[u(c_\tau) - \psi(K_{A_\tau})\,h(S_\tau) - Q(A_\tau)]\,e^{-z(t,\tau)}dt \tag{27}$$

Assumption 2 :

(i) We assume a twice continously differentiable utility function with following properties ; $\psi(K_A) > 0$, $\psi^{'}(K_A) < 0$, $\psi^{''}(K_A) > 0$.

(ii) When there does not exist any adaptation policy, we have $\psi(0) = \underline{\psi}$. Similar to Bréchet et al. (2012), we assume also $\psi(\infty) = \bar{\psi}$.

(iii) $Q(A) > 0$, $Q^{'}(A) > 0$, $Q^{''}(A) > 0$ and $Q(0) = 0$.

When society invests in adaptation capital, the impact of penalty due to catastrophic event is alleviated. This kind of modelling adaptation in line with Zemel (2015) and Tsur and Zemel (2015) differs from Bréchet et al. (2013) and Ayong Le Kama and Pommeret (2015) where the adaptation capital affects directly the damage function for all time. The adaptation capital in our model enters in the penalty which plays a role after the occurence of a catastrophic event. This specification is putting into the forefront of uncertainty concerning the accumulation of adaptation capital. The adaptation capital follows the dynamics ;

$$\dot{K}_A = A - \delta K_A \tag{28}$$

where δ is the scrap value for adaptation capital. At each moment, society decides optimally for its adaptation decisions. The Hamiltonian of the benchmark model becomes ;

$$\mathcal{H} = \frac{u(c) - \psi(K_A)\,h(S) - Q(A)}{\theta(S)} + \lambda\left[\frac{R(S) - c}{\theta(S)}\right] + \mu\left[\frac{A - \delta K_A}{\theta(S)}\right] \tag{29}$$

where λ is the shadow price for environmental quality (or renewable resources) and μ is the shadow price of adaptation capital. The optimal solution of the program is as follows ;

$$\frac{\partial \mathcal{H}}{\partial c} = \frac{u^{'}(c)}{\theta(S)} - \frac{\lambda}{\theta(S)} = 0 \tag{30}$$

$$\frac{\partial \mathcal{H}}{\partial A} = -\frac{Q^{'}(A)}{\theta(S)} + \frac{\mu}{\theta(S)} = 0 \tag{31}$$

$$\dot{c} = -\frac{u^{'}(c)}{u^{''}(c)}\left[R^{'}(S) - \theta(S) - \frac{\psi(K_A)\,h^{'}(S)}{u^{'}(c)} - \frac{\theta^{'}(S)}{\theta(S)}\left[\frac{u(c) - \psi(K_A)\,h(S) - Q(A)}{u^{'}(c)} + \frac{Q^{'}(A)}{u^{'}(c)}\dot{K}_A + \dot{S}\right]\right] \tag{32}$$

$$\dot{A} = \frac{Q^{'}(A)}{Q^{''}(A)}\left[\theta(S) + \delta + \frac{\psi^{'}(K_A)\,h(S)}{Q^{'}(A)}\right] \tag{33}$$

where two equations represents the first order conditions for consumption and adaptation investment and $\dot{c}$ and $\dot{A}$ stand for the optimal dynamics of consumption and adaptation investment.

It is interesting to analyze the effects of adaptation capital in this current economy concerning its effects on steady state.

Proposition 2. *(i) Adaptation policy could cause multiple equilibria in an economy where it could trap countries with high marginal utility of consumption to a lower equilibrium, by decreasing the environmental quality and increasing the probability of catastrophic event.*

(ii) Consequently, an economy without multiple equilibria at the beginning could face to a multiple equilibria with implementing only adaptation policy.

At steady state, from the dynamics of adaptation investment, we can write

$$-\psi^{'}(K_A) = Q^{'}(\delta K_A)\frac{(\rho + h(S) + \delta)}{h(S)} \tag{34}$$

. For this purpose, by the means of implicit function theorem, we write down the effect of adaptation capital on environmental quality at the steady state.

$$\frac{dK_A}{dS} = \frac{-h^{'}(S)\left(Q^{'}(\delta K_A) + \psi^{'}(K_A)\right)}{(\theta(S)+\delta)\, Q''(\delta K_A) + \psi''(K_A)\, h(S)} = \frac{h^{'}(S)\, Q^{'}(\delta K_A)\left(\frac{\rho+\delta}{h(S)}\right)}{(\theta(S)+\delta)\, Q''(\delta K_A) + \psi''(K_A)\, h(S)} < 0 \tag{35}$$

where the nominator and denominator are always negative and positive respectively. We can conclude from this relation that when the environmental quality is sufficiently high, the necessary level of adaptation capital K_A is less. This one is due to hazard rate which would be low with the higher environmental quality. Higher the environmental quality, lower the necessary level for adaptation capital K_A at the steady state. This relation has another economic interpretation. When people in society adapts more with a higher hazard rate, the negative consequences of penalty due to a possible catastrophe decreases, which could give an incentive to exploit more renewable resources more as people can assume to adapt more easily to the possible penalty.

As made in previous section, we define $G(S)$ function with adaptation capital ;

$$G(S) = R^{'}(S) - \theta(S) - \frac{\psi(K_A)\, h^{'}(S)}{u'(R(S))} - \frac{\theta^{'}(S)}{\theta(S)}\left[\frac{u(R(S)) - \psi(K_A)\, h(S)}{u'(R(S))}\right] \tag{36}$$

We write the condition for multiple equilibria in order to see the effect of adaptation capital in order to see analytically that it increases the possibility to cause multiple equilibria ;

$$G^{'}(S) = R^{''}(S) - \theta^{'}(S) - \left[\frac{\theta^{''}(S)\,\theta(S) - \left(\theta^{'}(S)\right)^2}{(\theta(S))^2}\right]\frac{u(c) - \psi(K_A)\, h(S) - Q(A)}{u'(c)} - \frac{\psi(K_A)\, h^{''}(S)}{u'(c)} \tag{37}$$

$$+\frac{\psi(K_A)\, u^{''}(c)}{(u'(c))^2} - \left[\frac{\theta^{'}(S)}{\theta(S)}\left(1 - \frac{(u(c) - \psi(K_A)\, h(S) - Q(A))\, u^{''}(c)}{(u'(c))^2}\right)\right]\left[R^{'}(S)\right]$$

$$+\underbrace{\left[\frac{\theta^{'}(S)}{\theta(S)}\left[\frac{\psi^{'}(K_A)}{u'(c)}(h(S) - \theta(S)) + \delta Q^{'}(\delta K_A)\right]\right]\frac{dK_A}{dS}}_{>0} > 0$$

As shown in this condition, the adaptation investment increases the possibility of a multiple equilibria which induces environmental inequalities accross countries. Consequently, an economy without implementing any policy which does not face to multiple equilibria could have a multiple equilibria.

The reason behind this environmental trap lays down on the fact that adaptation capital decreases the environmental quality, which causes an increase in hazard rate. When there are some countries with higher marginal utility, a social planner which devotes some part of its financial resources to adaptation investment will sacrifice a part of the consumption. It is evident that opportunity cost of sacrificing one unit of additional consumption is much important for those who have higher marginal utility of consumption. To understand better this fact, we rewrite the Euler equation as in the previous section in the following way ;

$$\dot{c} = -\frac{u^{'}(c_t)}{u^{''}(c_t)}\left[R^{'}(S_t) - \theta(S_t) - \frac{\psi(K_{A_t})\, h^{'}(S_t)}{u^{'}(c_t)} - \frac{\theta^{'}(S_t)}{u^{'}(c_t)}\int_t^{\infty}(u(c_\tau) - \psi(K_A)\, h(S_\tau) - Q(A_\tau))\, e^{-\Omega_\tau} d\tau\right] \tag{38}$$

As in countries where marginal utility of consumption is high, the variation in hazard is less important concerning the trade-off between catastrophic event and consumption. (see last expression of equation (38))

This does not mean that hazard rate is not playing a role for the trapped countries. On the contrary, the catastrophic event probability itself causes the multiple equilibria phenomena.

In case where an economy only adapts to climate change and not mitigating at all (in order to make its environmental conditions better off.) devotes some part of its financial resources for adaptation investment. Consequently, countries having maintaining a lower consumption level as they have invested in adaptation capital could have a higher marginal utility. In this case, these countries will be more insensitive to the changes of environmental quality which explains why investing only adaptation capital in countries with already higher level of marginal utility of consumption could trap to the lower equilibrium.

We make an illustration of $G(S)$ function that we have defined in the previous section with adaptation policy[12].

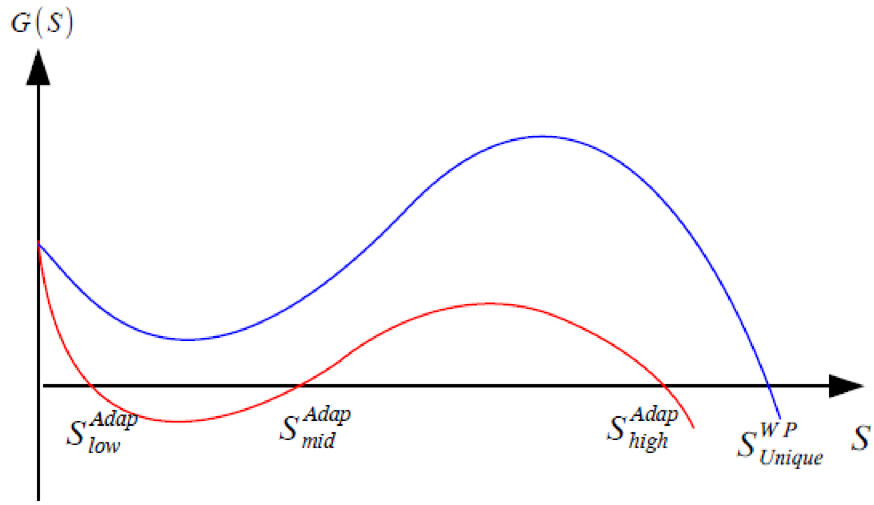


Figure 5 : $G(S)$ function without any policy and with only adaptation policy. (Blue curve represents $G(S)$ function for model without any policy and Red curve is the model with adaptation policy.)

As it can be seen by the multiple equilibria condition, making only adaptation policy could cause environmental inequalities across some regions in world due to the uncertain catastrophic event. Countries with sufficiently high levels of environmental quality are better off than countries with higher marginal utility of consumption (especially poor countries) because of the fact that their discount rate will be less higher than poor countries. As a nutshell, for these countries, having a higher environmental quality could provide much utility according to poor countries where the consumption level is close to its subsistence level and satisfying essential needs are providing more utility than accumulating a bigger resource stock, as those agents are impatient due to their high level of marginal utility of consumption. As mentionned above, they are less sensitive to changes in hazard rate. Note also that this configuration with multiple equilibria disappears in a model without uncertainty as the mechanism that is triggered with hazard rate disappears.

In order to see how the location of steady state curve $\dot{S} = 0$ changes in the phase diagram according to adaptation capital, we write by the means of implicit function theorem ;

$$\frac{dK_A}{dS} = -\frac{R''(S) - \theta'(S) - \Lambda(S)\frac{(u(R(S)) - \psi(K_A)h(S))}{u'(R(S))} - \frac{\psi(K_A)h''(S)}{u'(R(S))} + \frac{\psi(K_A)u''(R(S))}{\left(u'(R(S))\right)^2} - \left[\frac{\theta'(S)}{\theta(S)}\chi(S)\right]\left[R'(S)\right]}{\left[\frac{\theta'(S)}{\theta(S)}\left[\frac{\psi'(K_A)}{u'(R(S))}(h(S) - \theta(S)) + \delta Q'(\delta K_A)\right]\right]} < 0 \tag{39}$$

[12] The following graphic is also obteined also by a numerical analysis that we will present in the Numerical Analysis section with its detail.

where $\chi(S) = \left(1 - \frac{(u(R(S)) - \psi(K_A)h(S))u''(R(S))}{\left(u'(R(S))\right)^2}\right)$ and $\Lambda(S) = \left[\frac{\theta''(S)\theta(S) - \left(\theta'(S)\right)^2}{(\theta(S))^2}\right]$.

We know that the denominator is always negative. However the denominator is negative for low and high steady-state but negative for middle steady-state. As the middle steady-state, the environmental quality increases with the adaptation capital but as this equilibrum is an unstable equilibrium, this effect is a transitory one. When the regeneration of environment is high, even with accumulation of adaptation capital, the steady-state level of resource stock could be higher. Instantly, higher regeneration could dominate the effect of adaptation capital on environmental quality.

We can represent the steady-state curves in order to illustrate how only implementing an adaptation policy could cause environmental inequalities across different countries or regions ;

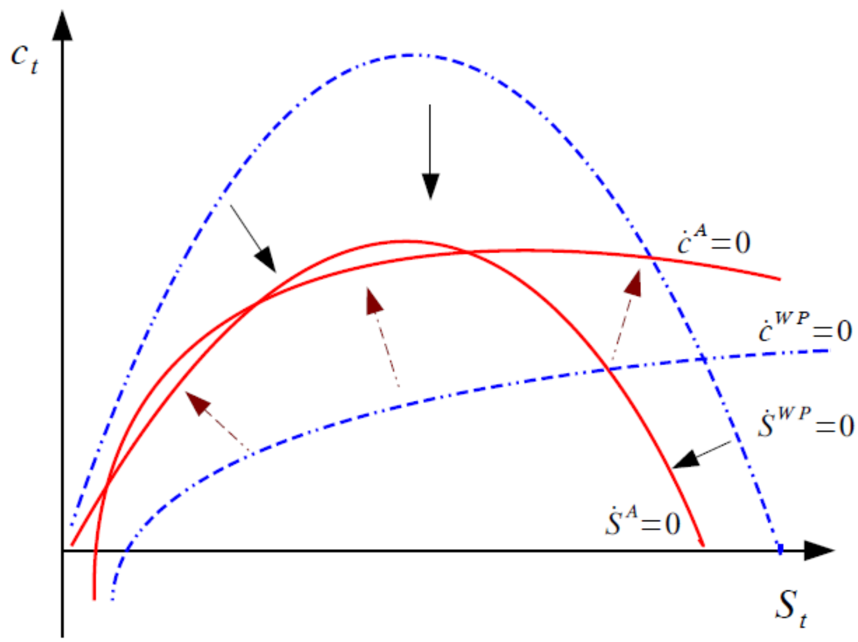


Figure 6 : Phase diagram (Blue dashed curves represents the model without any policy and red thick curves stand for the economy with only adaptation policy.)

We show graphically that steady state curve of environmental quality $\dot{S} = 0$ shifts downwards with the adaptation capital due to decreased environmental quality. Following this effect, society will be more impatient and will tend to increase its consumption level, which will result in a upward shift of steady state curve of consumption $\dot{c} = 0$.

4 Model with Mitigation Policy

In this section, we analyze the benchmark model with mitigation policy and show how qualititative results could change according to the original problem and in which cases this policy could escape a country from a low consumption and low environmental quality steady state. The social planner seeks to solve the following maximization problem, similar to the first problem ;

$$\underset{c_t}{max} E_t \left[\int_\tau^T (u(c_t) - P_M M_t) e^{-\rho(\tau - t)} dt + \varphi(S_T) e^{-\rho(T-t)}\right]$$
$$= \int_t^\infty \left[(u(c_\tau) - P_M M_\tau) \frac{(1 - F(\tau))}{(1 - F(t))} + \frac{f(\tau)}{(1 - F(t))}\varphi(S_\tau)\right] e^{-\rho(\tau - t)} d\tau \qquad (40)$$

where P_M is the unit cost for mitigation policy. In this model, contrary to the model with adaptation policy, mitigation activities are taken as a flow variable similar to Smulders and Gradus (1993) and Bréchet et al. (2013). The form of the post value function changes and takes the form as in the benchmark model ;

$$\varphi(S) = \int_0^{\infty} u(c_{min}) e^{-\rho t} dt - \psi = -\psi \tag{41}$$

By integrating by parts, the social planner's maximization program becomes ;

$$\max_{c_t} \int_t^{\infty} [u(c_\tau) + h(S_\tau)\varphi(S_\tau) - P_M M] e^{-z(t,\tau)} dt = \int_t^{\infty} [u(c_\tau) - \psi h(S_\tau) - P_M M_\tau] e^{-z(t,\tau)} dt \tag{42}$$

Assumption 3 :

(i) We assume a twice continously differentiable, increasing and concave mitigation function with following properties ; $\Gamma(M) > 0$, $\Gamma^{'}(M) > 0$, $\Gamma^{''}(M) < 0$.

(ii) When there does not exist any mitigation policy, we have $\Gamma(0) = 0$.

In this section, the environmental quality evolves as

$$\dot{S}_t = R(S_t) - c_t + \Gamma(M_t) \tag{43}$$

where mitigation activities increases the quality of environment as expected. The Hamiltonian of the benchmark model becomes ;

$$\mathcal{H} = \frac{u(c) - \psi h(S) - Q(A)}{\theta(S)} + \lambda \left[\frac{R(S) - c + \Gamma(M)}{\theta(S)} \right] \tag{44}$$

where λ is the shadow price for environmental quality (or renewable resources). The optimal solution of the program is as follows ;

$$\frac{\partial \mathcal{H}}{\partial c} = \frac{u^{'}(c)}{\theta(S)} - \frac{\lambda}{\theta(S)} = 0 \tag{45}$$

$$\frac{\partial \mathcal{H}}{\partial A} = -\frac{P_M}{\theta(S)} + \frac{\lambda \Gamma^{'}(M)}{\theta(S)} = 0 \tag{46}$$

$$\dot{c} = -\frac{u^{'}(c)}{u^{''}(c)} \left[R^{'}(S) - \theta(S) - \frac{\psi h^{'}(S)}{u^{'}(c)} - \frac{\theta^{'}(S)}{\theta(S)} \left[\frac{u(c) - \psi h(S) - P_M M}{u^{'}(c)} + \dot{S} \right] \right] \tag{47}$$

$$\dot{M} = \frac{\Gamma^{'}(M)}{\Gamma^{''}(M)} \left[R^{'}(S) - \theta(S) - \frac{\psi h^{'}(S)}{u^{'}(c)} - \frac{\theta^{'}(S)}{\theta(S)} \left[\frac{u(c) - \psi h(S) - P_M M}{u^{'}(c)} + \dot{S} \right] \right] \tag{48}$$

where two equations represents the first order conditions for consumption and mitigation activities and $\dot{c}$ and $\dot{M}$ stand for the optimal dynamics of consumption and mitigation activities.

Proposition 3. *(i) Mitigation policy could save an economy from multiple equilibria where there are countries stuck at a low environmental quality and low consumption steady state, by increasing the environmental quality and decreasing the probability of catastrophic event.*

(ii) Consequently, an economy with multiple equilibria at the beginning could escape multiple equilibria with implementing mitigation policy.

At steady state, from the first order condition for mitigation activities ;

$$P_M = u^{'}[R(S) + \Gamma(M)] \Gamma^{'}(M) \tag{49}$$

where $c = R(S) + \Gamma(M)$ at the steady state. By differenciating totally the optimality condition at steady state, we can find how optimal mitigation level changes according to environmental quality at steady state.

$$\frac{dM}{dS} = -\frac{u^{''}(c) R^{'}(S) \Gamma^{'}(M)}{u^{''}(c)(\Gamma^{'}(M))^{2} + u^{'}(c)\Gamma^{''}(M)} < 0 \tag{50}$$

We can conclude that when the environmental quality increases, the necessary level of abatement decreases, which is a quite intuitive result. We write the sufficiency condition for multiple steady state ;

$$G^{'}(S) = R^{''}(S) - \theta^{'}(S) + \left[\frac{\psi u^{''}(c) h^{'}(S)}{(u^{'}(c))^{2}} - \frac{\theta^{'}(S)}{\theta(S)}\left(1 - \frac{u^{''}(c)(u(c) - \psi h(S) - P_M M)}{(u^{'}(c))^{2}}\right)\right]\left[R^{'}(S)\right] - \frac{\psi h^{''}(S)}{u^{'}(c)}$$

$$-\Lambda(S)\left[\frac{u(c) - \psi h(S) - P_M M}{u^{'}(c)}\right] + \underbrace{\left[\frac{\psi u^{''}(c) h^{'}(S)}{(u^{'}(c))^{2}} + \frac{\theta^{'}(S)}{\theta(S)}\left(\frac{u^{''}(c)(u(c) - \psi h(S) - P_M M)}{(u^{'}(c))^{2}}\right)\right]\Gamma^{'}(M)\frac{dM}{dS}}_{<0} > 0 \tag{51}$$

We can observe easily that mitigation activities decreases the possibility to have a multiple equilibria economy. Consequently, an economy suffering from multiple equilibria could escape multiple equilibria with implementing a mitigation policy. As mitigation activities increases environmental quality, the hazard rate decreases, which means that there are less chances that we have a multiple equilibria economy. Mitigation activitiy allows an economy with high marginal utility of consumption to escape a low steady state because it increases the possibilty of a higher consumption level at the phase of transition to steady state, which permit agents to fulfill their basic needs. Especially, this is crucial for countries in which the most of economic activities are based on agriculture and exploitation of natural resources.

Also, with accumulating much more resource stock with mitigation activities, satiation of utility in poor countries let them to accumulate much more environmental capital, as it is providing more utility than directing all consumption today.

In order to see how mitigation activity could save an economy from multiple equilibria, we rewrite the Euler equation ;

$$\dot{c} = -\frac{u^{'}(c_t)}{u^{''}(c_t)}\left[R^{'}(S_t) - \theta(S_t) - \frac{\psi h^{'}(S_t)}{u^{'}(c_t)} - \frac{\theta^{'}(S_t)}{u^{'}(c_t)}\int_t^{\infty}(u(c_\tau) - \psi h(S_\tau) - P_M M_\tau)e^{-\Omega_\tau}d\tau\right] \tag{52}$$

When an economy performs mitigation activities, it decreases the hazard rate, also with increasing the consumption possibilities at transition phase to steady state. Then, we observe that with mitigation activity, last term increases, which is due to fact that consumption level of poor countries would not be close to subsistence level. In this case, society would be more sensitive to changes in environmental conditions, which will let them to be able to postpone their consumption, with increasing the resource stock.

We make an illustration of $G(S)$ function with mitigation policy ;

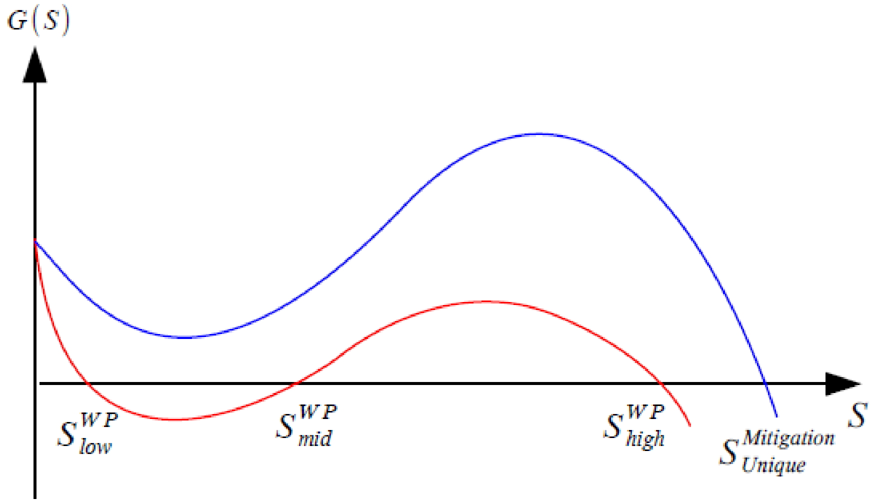


Figure 7 : $G(S)$ function without any policy and with only mitigation policy. (Blue curve represents $G(S)$ function for model with only mitigation policy and red curve is the model without policy.)

An economy suffering from environmental inequalities could avoid this phenomena with implementing mitigation policy, by decreasing directly hazard rate of catastrophic event. The risk reduction plays a crucial role for the reason that people could make further plans for future, as the survival rate would be higher with mitigation policy.

We can easily show on a phase diagram how mitigation activities could save an economy from environmental inequalities ;

$$\frac{dM}{dS} = -\frac{R^{''}(S) - \theta^{'}(S) + \upsilon(c,S)\left[R^{'}(S)\right] - \frac{\psi h^{''}(S)}{u^{'}(c)} - \Lambda(S)\left[\frac{u(c)-\psi h(S)-P_M M}{u^{'}(c)}\right]}{\left[\frac{\psi u''(c)h'(S)}{\left(u'(c)\right)^2} + \frac{\theta'(S)}{\theta(S)}\left(\frac{u''(c)(u(c)-\psi h(S)-P_M M)}{\left(u'(c)\right)^2}\right)\right]\Gamma'(M)} > 0 \tag{53}$$

where $\upsilon(c,S) = \left[\frac{\psi u^{''}(c)h^{'}(S)}{\left(u^{'}(c)\right)^2} - \frac{\theta^{'}(S)}{\theta(S)}\left(1 - \frac{u^{''}(c)(u(c)-\psi h(S)-P_M M)}{\left(u^{'}(c)\right)^2}\right)\right]$. We can see by this relation that mitigation activities are increasing the steady state level of environmental quality, which is equivalent to an upward shift of the steady state curve of environmental quality.

As by reasons mentionned aformentionned, with the increased environmental quality, agents could postpone their consumption with the decreased hazard rate, which would shift downwards the steady state curve of consumption.

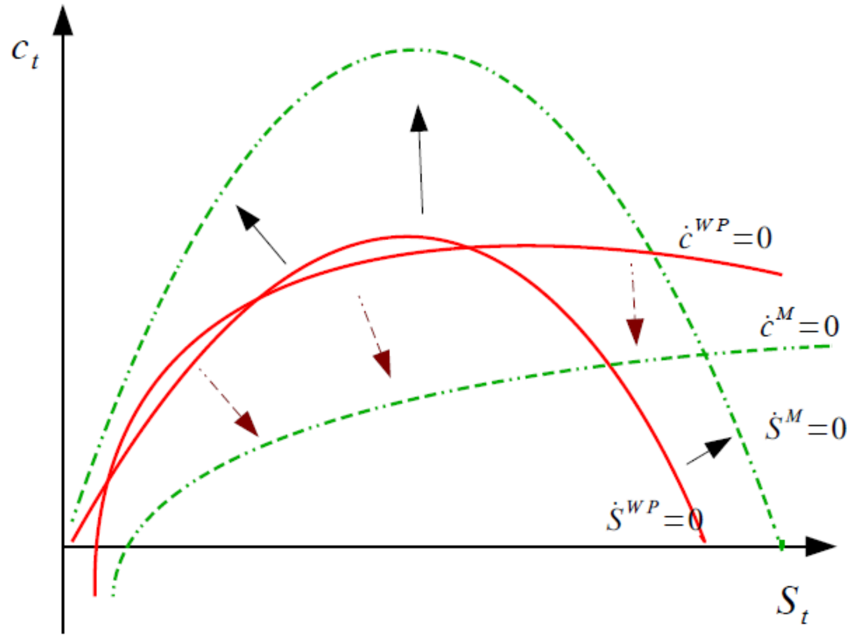


Figure 8 : Phase diagram on a plane (c, S) (Green curves represents the model with mitigation policy red curves are for the model without policy.)

We show graphically that steady state curve of environmental quality $\dot{S} = 0$ shifts upward with mitigation due to increased environmental quality. Following this effect, society will be more patient and will tend to postpone consumption, which will result in a downward shift of steady state curve of consumption $\dot{c} = 0$.

5 Model with Adaptation and Mitigation Policy

In this section, we analyze the full model with adaptation and mitigation policy and analyze how an optimal policy mix of adaptation and mitigation should be implemented in order to avoid an economy with environmental inequalities across countries and regions.

As we have showed in the last two previous two sections, adaptation and mitigation policies create opposite effects concerning the possibility of multiple equilibria, former one increasing and latter decreasing this possibility. In this case, there is also a trade-off between these two policies. At one hand, only adaptation could cause inequalities as shown above by increasing hazard rate, on the other hand only mitigation could save economy from inequalities by decreasing the probability of catastrophe. This does not mean that adaptation investment is unncessary. In the case of catastrophic event, economy will face a severe penalty due to catastrophic event, in which case agents must cope with this penalty. Consequently, the optimal policy mix must be decided with taking into account these opposite effects but it should be implemented in a way that avoids multiple equilibria economy.

The social planner seeks to solve the following maximization problem, similar to the first problem ;

$$\underset{c_t}{max} E_t \left[\int_{\tau}^{T} \left(u\left(c_t\right) - P_M M_t - Q\left(A_t\right) \right) e^{-\rho(\tau - t)} dt + \varphi\left(S_T, K_{A_T}\right) e^{-\rho(T-t)} \right]$$

$$= \int_{t}^{\infty} \left[\left(u\left(c_\tau\right) - P_M M_\tau - Q\left(A_\tau\right) \right) \frac{\left(1 - F\left(\tau\right)\right)}{\left(1 - F\left(t\right)\right)} + \frac{f\left(\tau\right)}{\left(1 - F\left(t\right)\right)} \varphi\left(S_\tau, K_{A_\tau}\right) \right] e^{-\rho(\tau - t)} d\tau \qquad (54)$$

where $H(A)$ stands for the cost function of adaptation investment as a function of adaptation investment A like in the previous section. In this model with adaptation capital K_A, the form of the post value function takes the form ;

$$\varphi(S, K_A) = \int_0^\infty u(c_{min}) e^{-\rho t} dt - \psi(K_A) = -\psi(K_A) \tag{55}$$

By integrating by parts the equation, maximization program becomes ;

$$\underset{c_t}{max} \int_t^\infty \left[u(c_\tau) + h(S_\tau)\varphi(S_\tau) - P_M M_\tau - Q(A_\tau)\right] e^{-z(t,\tau)} dt$$

$$= \int_t^\infty \left[u(c_\tau) - \psi(K_A) h(S_\tau) - P_M M_\tau - Q(A_\tau)\right] e^{-z(t,\tau)} dt \tag{56}$$

subject to

$$\dot{K}_{A_t} = A_t - \delta K_{A_t} \tag{57}$$

and

$$\dot{S}_t = R(S_t) - c_t + \Gamma(M_t) \tag{58}$$

The Hamiltonian that social planner solves ;

$$\mathcal{H} = \frac{u(c) - \psi(K_A) h(S) - P_M M - Q(A)}{\theta(S)} + \lambda \left[\frac{R(S) - c + \Gamma(M)}{\theta(S)}\right] + \mu \left[\frac{A - \delta K_A}{\theta(S)}\right] \tag{59}$$

where λ represents the shadow price for environmental quality (or renewable resources) as in the previous sections and μ is the shadow price of adaptation capital. The optimal solution of the program is as follows ;

$$\frac{\partial \mathcal{H}}{\partial c} = \frac{u^{'}(c)}{\theta(S)} - \frac{\lambda}{\theta(S)} = 0 \tag{60}$$

$$\frac{\partial \mathcal{H}}{\partial A} = -\frac{Q^{'}(A)}{\theta(S)} + \frac{\mu}{\theta(S)} = 0 \tag{61}$$

$$\frac{\partial \mathcal{H}}{\partial M} = -\frac{P_M}{\theta(S)} + \frac{\lambda \Gamma^{'}(M)}{\theta(S)} = 0 \tag{62}$$

$$\dot{c} = -\frac{u^{'}(c)}{u''(c)} \left[R^{'}(S) - \theta(S) - \frac{\psi(K_A) h^{'}(S)}{u^{'}(c)} - \frac{\theta^{'}(S)}{\theta(S)} \left[\frac{u(c) - \psi(K_A) h(S) - P_M M - Q(A)}{u^{'}(c)} + \frac{Q^{'}(A)}{u^{'}(c)} \dot{K}_A + \dot{S}\right]\right] \tag{63}$$

$$\dot{A} = \frac{Q^{'}(A)}{Q''(A)} \left[\theta(S) + \delta + \frac{\psi^{'}(K_A) h(S)}{Q^{'}(A)}\right] \tag{64}$$

$$\dot{M} = \frac{\Gamma^{'}(M)}{\Gamma''(M)} \left[R^{'}(S) - \theta(S) - \frac{\psi(K_A) h^{'}(S)}{u^{'}(c)} - \frac{\theta^{'}(S)}{\theta(S)} \left[\frac{u(c) - \psi(K_A) h(S) - P_M M - Q(A)}{u^{'}(c)} + \frac{Q^{'}(A)}{u^{'}(c)} \dot{K}_A + \dot{S}\right]\right] \tag{65}$$

where first three equations represents the first order conditions for consumption, adaptation and mitigation activities respectively and $\dot{c}$, $\dot{A}$ and $\dot{M}$ stand for the optimal dynamics of consumption and mitigation activities.

Proposition 4. *(i) An economy can avoid multiple equilibria with a policy mix of adaptation and mitigation, depending on how much of the total income is splitted between adaptation capital and mitigation activity. If economy invests so much in adaptation capital than in mitigation, the possibility to having multiple equilibria increases.*

Both of the adaptation and mitigation activities are helpful for society and increases the welfare of agents, first one by decreasing the severity of the penalty that occurs after catastrophe and latter by decreasing the hazard rate and increasing both environmental stock and consumption. However, a society must decide carefully how it must invest in adaptation policy. There is no doubt that society must take all precaution to be able to adapt to consequences of a possible catastrophe but we should not forget that adaptation policy, as shown above, could create an incentive to use more stock of resource. This one is plausible because agents are able to adapt to climate change and investing only in adaptation policy without doing or doing less mitigation relatively to adaptation, could increase more the environmental problems in a world that we face catastrophic events.

Recent IPCC report - Climate Change 2014: Impacts, Adaptation, and Vulnerability (2014) are highlighting the fact that society should invest heavily on adaptation activities without even mentionning the effects shown above. The view expressed in this report claims that even if develeoped countries take the necessary measures in order to cut their emissions, the positive effects of this action will appear after some decades. Another strong claim of this report is that developping countries are mostly vulnerable to climate change more for other reasons than greenhouse gas emissions.

We argue basically that only adaptation to climate change is not an ultime solution contrary to this report. In a world with uncertainty, implementing adaptation policy in developping countries are even more problematic than in developped countries. Firstly, we have shown that implementing an adaptation policy increases the hazard rate of a catastrophe, which causes a multiple equilibria economy in which there are countries suffering from low levels of consumption.

Consequently, making only adaptation policy in these countries where people are highly impatient, will make exhaust more the natural resource stock from where a biggest part of their income is coming. When there exists only an adaptation policy, agents will start to consume much. As the natural stock is already not sufficiently built (or exhausted), the society starts with an higher hazard rate, which will push the economy use a bigger part of the resource, which case result in a vicious cycle that poor countries will trap in an equilibrium with low consumption and environmental stock level. That can be alternatively named as an "environmental trap".

Secondly, we argue that the idea of cutting emissions will have its benefits afterwards is dangerous. This kind of statement will probably increase the environmental and income inequalities in a world with catastrophic events. We have stated that most of Sub Saharan African countries income is stemming from natural resources. Even by adapting to climate change, not mitigating or giving less importance to mitigation will affect more these countries as we have shown that inequalities arises with high levels of catastrophic event probability. The statements of adapting more to climate change and mitigating less will increase without doubt the catastrophic event probability, which will by default increase inequalities between countries, trapping some regions and countries to low consumption and environmental quality levels, which case will trigger catastrophic events.

We write the sufficiency condition for multiple steady state ;

$$
\begin{aligned}
G^{'}(S) = R^{''}(S) - \theta^{'}(S) - \left[\frac{\theta^{''}(S)\,\theta(S) - \left(\theta^{'}(S)\right)^{2}}{\left(\theta(S)\right)^{2}}\right]\left[\frac{u(c) - \psi(K_A)\,h(S) - Q(A) - P_M M}{u^{'}(c)}\right] \\
+\frac{\psi(K_A)\,u^{''}(c)}{\left(u^{'}(c)\right)^{2}} - \left[\frac{\theta^{'}(S)}{\theta(S)}\left(1 - \frac{\left(u(c) - \psi(K_A)\,h(S) - Q(A) - P_M M\right)u^{''}(c)}{\left(u^{'}(c)\right)^{2}}\right)\right]\left[R^{'}(S)\right] \\
-\frac{\psi(K_A)\,h^{''}(S)}{u^{'}(c)} + \underbrace{\left[\frac{\theta^{'}(S)}{\theta(S)}\left[\frac{\psi^{'}(K_A)}{u^{'}(c)}\left(h(S) - \theta(S)\right) + \delta Q^{'}(\delta K_A)\right]\right]\frac{dK_A}{dS}}_{>0}
\end{aligned}
$$

$$+\underbrace{\left[\frac{\psi u^{''}(c)\,h^{'}(S)}{\left(u^{'}(c)\right)^{2}}+\frac{\theta^{'}(S)}{\theta(S)}\left(\frac{u^{''}(c)\left(u(c)-\psi h(S)-Q(A)-P_M M\right)}{\left(u^{'}(c)\right)^{2}}\right)\right]\Gamma^{'}(M)\frac{dM}{dS}}_{<0}>0 \tag{66}$$

We can also see the presence of trade-off between adaptation and mitigation mathematically on the sufficiency condition for multiple equilibria. At one hand, mitigation increases the possibility of a unique equilibrium, on the other hand adaptation increases the possibility of multiple equilibria as it increases the hazard rate of catastrophic event. Having multiple or unique equilibria depends on which policy will implemented more. Analytically, we write down the following condition ;

$$\begin{cases} \frac{dM}{dK_A}>\frac{-\frac{\theta^{'}(S)}{\theta(S)}\left[\frac{\psi^{'}(K_A)}{u^{'}(c)}(h(S)-\theta(S))+\delta Q^{'}(\delta K_A)\right]}{\left[\frac{\psi u^{''}(c)h^{'}(S)}{\left(u^{'}(c)\right)^{2}}+\frac{\theta^{'}(S)}{\theta(S)}\left(\frac{u^{''}(c)\left(u(c)-\psi h(S)-Q(A)-P_M M\right)}{\left(u^{'}(c)\right)^{2}}\right)\right]\Gamma^{\prime}(M)}, & \text{Unique Equilibrium.}\\ \frac{dM}{dK_A}=\frac{-\frac{\theta^{'}(S)}{\theta(S)}\left[\frac{\psi^{'}(K_A)}{u^{'}(c)}(h(S)-\theta(S))+\delta Q^{'}(\delta K_A)\right]}{\left[\frac{\psi u^{''}(c)h^{'}(S)}{\left(u^{'}(c)\right)^{2}}+\frac{\theta^{'}(S)}{\theta(S)}\left(\frac{u^{''}(c)\left(u(c)-\psi h(S)-Q(A)-P_M M\right)}{\left(u^{'}(c)\right)^{2}}\right)\right]\Gamma^{\prime}(M)}, & \text{No Effect.}\\ \frac{dM}{dK_A}<\frac{-\frac{\theta^{'}(S)}{\theta(S)}\left[\frac{\psi^{'}(K_A)}{u^{'}(c)}(h(S)-\theta(S))+\delta Q^{'}(\delta K_A)\right]}{\left[\frac{\psi u^{''}(c)h^{'}(S)}{\left(u^{'}(c)\right)^{2}}+\frac{\theta^{'}(S)}{\theta(S)}\left(\frac{u^{''}(c)\left(u(c)-\psi h(S)-Q(A)-P_M M\right)}{\left(u^{'}(c)\right)^{2}}\right)\right]\Gamma^{\prime}(M)}, & \text{Multiple Equilibria.} \end{cases} \tag{1}$$

We can interpret this condition in the following way ; when economy accumulates so much adaptation capital than doing mitigation activities, it is more probable that multiple equilibria occurs due to an excessive increase in hazard rate with adaptation policy. The unique equilibrium is more possible when economy mitigates sufficiently, which would lead a decrease in hazard rate.

In other terms, in the case where the risk reduction effect due to mitigation activity dominates the increase of catastrophe proability from adaptation policy, the unique equilibrium possibility increases.

6 Numerical Illustration

In this section, we present a numerical analysis in order to show that multiple steady state possibility is not just a theoretical possibility. For this part, we use common functional specifications and calibration used in literature, similar to Bommier et al (2015) and Ren and Polasky (2014). The utility function that we choose is a CRRA function. The utility function takes the following form with including also the minimum level of consumption, which is supposed to be subsistence level consumption after the occurence of catastrophe ;

$$u(c)=\frac{c^{1-\sigma}-c_{min}^{1-\sigma}}{1-\sigma} \tag{67}$$

We take $c_{min}=1$ as in Bommier et al. (2015) and $\sigma=1.5$ for the intertemporal elasticity of substitution. As in Ren and Polasky (2014), we take a logistic growth function for the environmental quality (or renewable resource stock) ;

$$R(S)=S+gS\left(1-\frac{S}{\bar{S}}\right) \tag{68}$$

where $\bar{S}=5$ is the carrying capacity of environment and $g=0.05$ is the intrinsic growth rate of the resource stock. We use the functional specification for hazard rate as in Ren and Polasky (2014) ;

$$h(S)=\frac{2\bar{h}}{1+exp\left[\eta\left(s/s^{*}-1\right)\right]} \tag{69}$$

where $\bar{h} = 0.5$ which is the upper bound for the hazard rate. The parameter η holds for the endogeneity level of catastrophic event. When this parameter equals to zero, it means that the catastrophic event is no more endogeneous and depends on exogeneous parameters. We set this parameters between 2 and 5 depending on the benchmark economy without policy when there are unique and multiple equilibria. s^* is the risk-free steady state of resource stock and calculated to be 52.45. The level of this parameter is found by simulating the concerning model with fixing $\bar{h} = 0$, in which current model reduces to a deterministic traditional growth model.

Given $\bar{h} \in [0, 1]$, when S is sufficiently high, the risk becomes very low depending also on level of endogeneity of catastrophic risk. The hazard rate takes the form ;

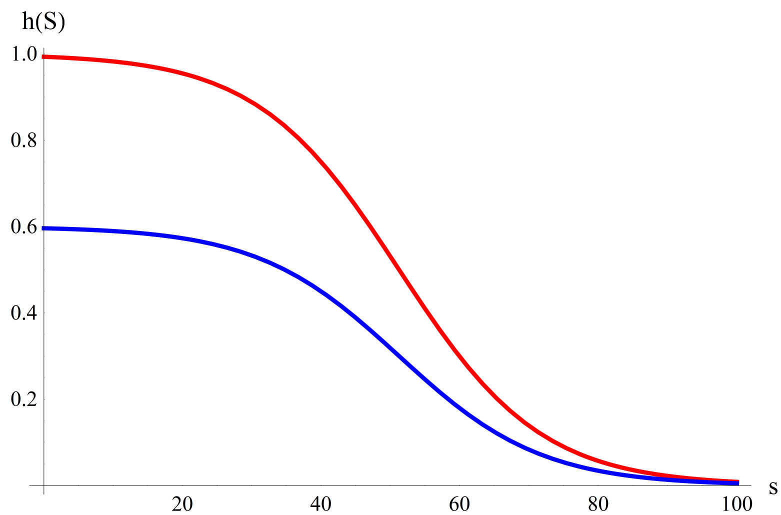


Figure 9 : Hazard rate for catastrophic risk (Red and blue lines hold for high and low risk profile respectively.)

For this graphic, the $\bar{h}$ is fixed to 0.25 for low risk profile while this value is held at 0.5 for high risk profile. For the rest of the numerical analysis, we will use this calibration when we compare our results concerning high and low risk profile economy. The cost function for adaptation investment is as follows ;

c

where $\phi_A = 0.5$ is the scale parameter of the marginal cost of adaptation investment. For adaptation function, we use the following form which is quite similar to Bréchet et al. (2012) ;

$$\psi(K_A) = \bar{\psi}\left(\omega + (1-\omega)e^{-\gamma K_A}\right) \tag{71}$$

where $\bar{\psi} = 10$ is the penalty rate where economy does not implement any adaptation policy. The parameters $\omega = 0.1$ and $\gamma = 0.6$ can be interpreted as efficiency parameters of adaptation activity. If ω is not sufficiently high, the adaptation capital is not much able to decrease the penalty rate. We γ can be considered as the marginal contribution of the adaptation capital to decrease the penalty amount. For the mitigation activity, which we consider as a flow variable, we use the function

$$\Gamma(M) = M^{\alpha} \tag{72}$$

which is a Cobb-Douglas function, which is also used in Ayong Le Kama and Pommeret (2014) but their model takes mitigation as a state variable different than our approach which is in the same line with Bréchet et al. (2012) for the specification of mitigation activity. The parameter $\alpha = 0.75$ is the elasticity of mitigation activity. The parameters for scrap value of adaptation capital, price of mitigation activity and the rate of pure time preference are fixed as $\delta = 0.065$, $P_M = 0.0005$ and $\rho = 0.025$.

Before starting analysis on numerical analysis, we find useful to show that model without any catastrophic risk reduces to simple deterministic growth model with unique equilibrium ;

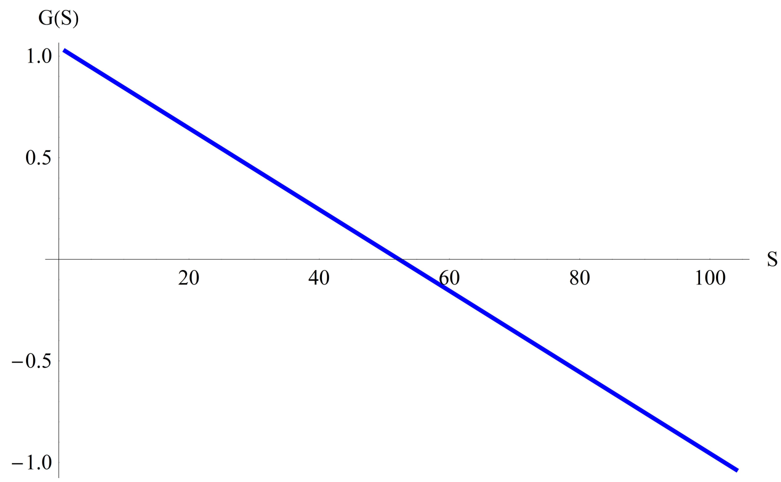


Figure 9.1 : $G(S)$ function without any catastrophic risk

We set $\bar{h} = 0$ in which case hazard function $h(S) = 0$ for all time. As we can see, the $G(S)$ function has one root, which means that deterministic model without any catastrophic risk has an unique equilibrium.

6.1 Adaptation Policy Causing Multiple Equilibria

For a numerical exercice, we suppose that an economy without any policy does not suffer from environmental inequalities. The only parameter that we change in our calibration that we use to get the following unique equilibrium economy is $\eta = 3.5$ We get the following $G(S)$ function ;

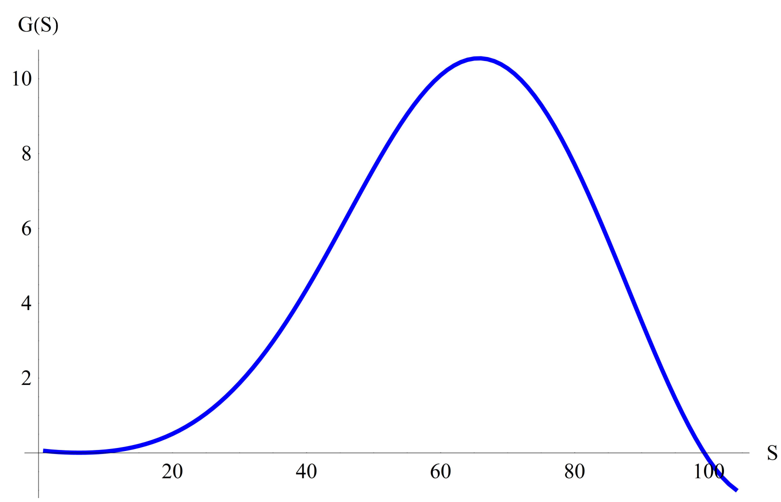


Figure 10 : $G(S)$ function without policy with unique equilibrium economy

While an economy does not face a higher risk, it can have a unique equilibrium. Note that the multiple equilibria arises with an higher endogeneity of catastrophe based on resource stock. In order to see more clearly that there exists only one equilibrium, we split the figure in two parts.

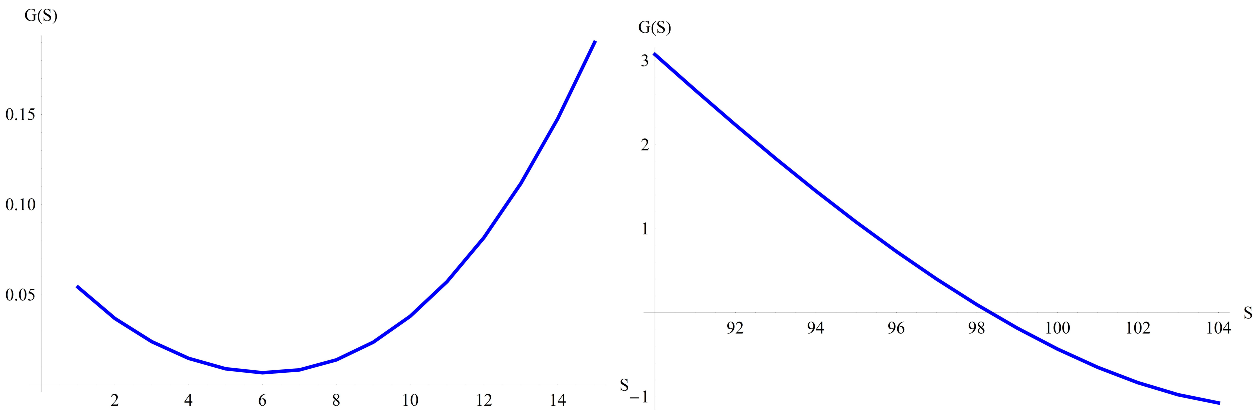


Figure 10.1 : $G(S)$ function splitted in two parts without policy with unique equilibrium economy

We now show that adaptation capital causes multiple equilibria in the above benchmark economy with unique equilibrium. As mentionned in previous sections, the multiple equilibria (i.e environmental inequalities) comes from the increased hazard rate due to excessive accumulation of adaptation capital.

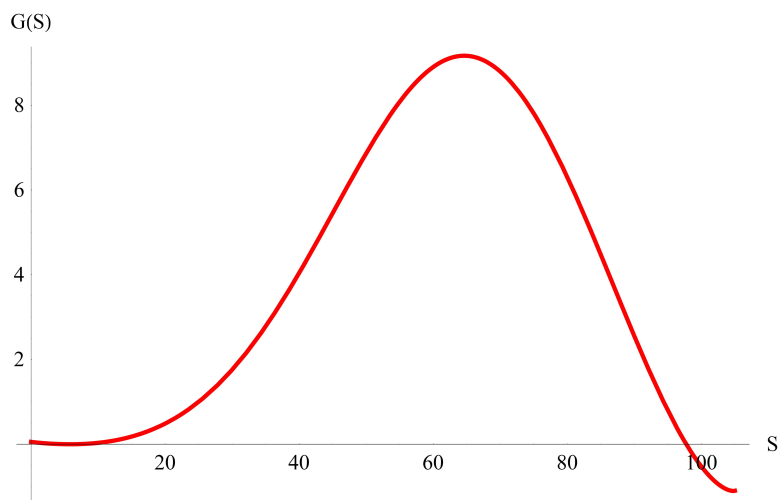


Figure 10.2 : $G(S)$ function for an economy implementing only adaptation policy

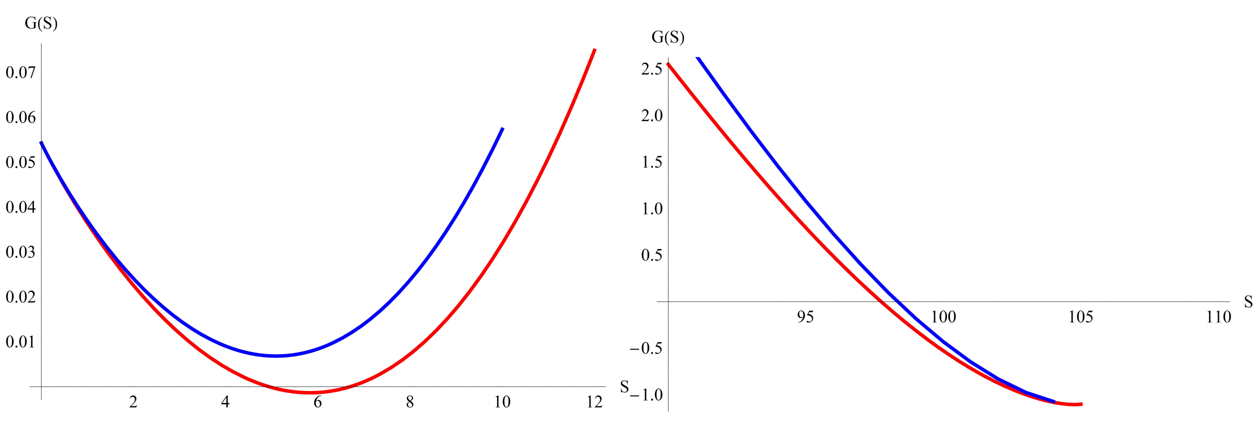


Figure 10.3 : $G(S)$ function for an economy implementing only adaptation policy causing multiple equilbria (Red and blue curves hold for model with adaptation policy and without policy respectively.)

When we look at the graphics, we see in the graphic on the left that adaptation policy (c.f red curves) causes multiple equilibria due to the increased hazard rate while the benchmark model has an unique equilibrium. On the graphic on the right side, we also see that the steady state resource stock with adaptation policy is less relatively to its level without policy. This means basically that even when the economy with adaptation policy converges to steady state with high level of consumption and resource stock, the resource stock is always lower, which case also strengthens our argument that adaptation policy must be coupled with an appropriate mitigation policy.

Sub Saharan African, Latin American and South Asian countries are said to be the most vulnerable regions in world against to climate change and most of policy makers suggestions are based on adapting more to changes in environmental conditions. (See IPCC Report on Adaptation Policy - 2014). What happens exactly for these regions of the world when there exists an adaptation capital ? At this purpose, in order to have better insights for this question, it is useful to see how the transitional dynamics are for adaptation capital accumulation for low and high steady state.

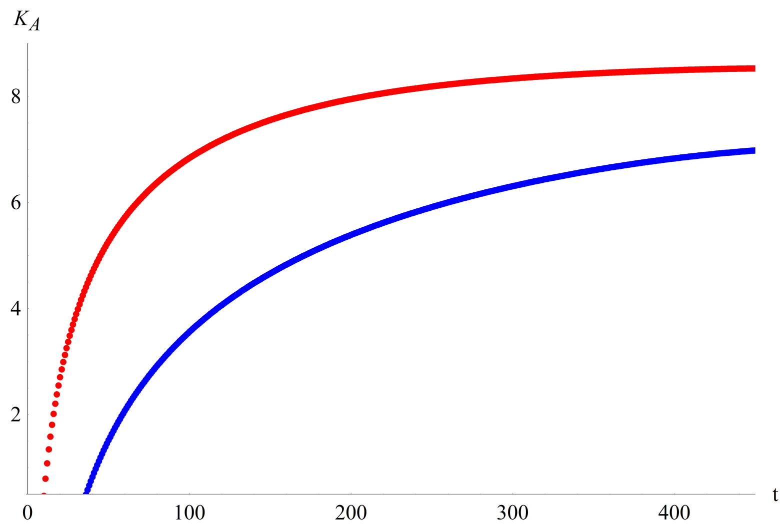


Figure 11 : Adaptation capital K_A accumulation (Red and blue path represent the optimal dynamics of capital accumulation for low and high steady state respectively)

We observe that economy that is trapped to low equilibrium accumulates more adaptation capital. This result is intuitive because as environmental quality is supposed to be lower in developping countries, there exists a higher adaptation capital accumulation in order to lower its impacts, which case increase hazard rate of catastrophe and traps some countries to lower again the stock of environmental quality. Wherefore, as the initial hazard rate is higher, these countries must accumulate more adaptation capital to decrease the amount of penalty due to a catastrophic event but note that these countries are trapping to low steady-state.

As can be seen on our analysis which shows that accumulating more adaptation capital implies a higher decrease in environmental quality. Therefore, a policy maker suggesting more adaptation capital for regions where resource stock is relatively at low levels will push developping countries to pollute more. Correspondingly, policy makers would suggest afterwards more and more to adapt to climate change. As a nutshell, all these elements would create a vicious cycle that we denote “more adaptation capital - less resource stock” effect. Wherefore, the frequency of catastrophic events will surely increase, which case would also cause important issues concerning development process of many countries across world.

6.2 Mitigation Policy Saving from Multiple Equilibria

In this section, we show numerically how mitigation policy could save an economy from multiple equilibria with increasing both environmental stock and consumption level at the steady state. The mechanism behind escaping multiple equilibria lays down firstly on decreasing the hazard rate of catastrophe, and secondly on giving the possibility of higher consumption possibilitys during the transition path to steady state. In this conditions, environmental and consumption inequalities between different countries or regions tend to disappear.

Firstly, we take an economy without environmental policy, which suffers from multiple equilibria. In order to have a multiple equilibria economy, we increase the endogeneity of catastrophic event η to 5.

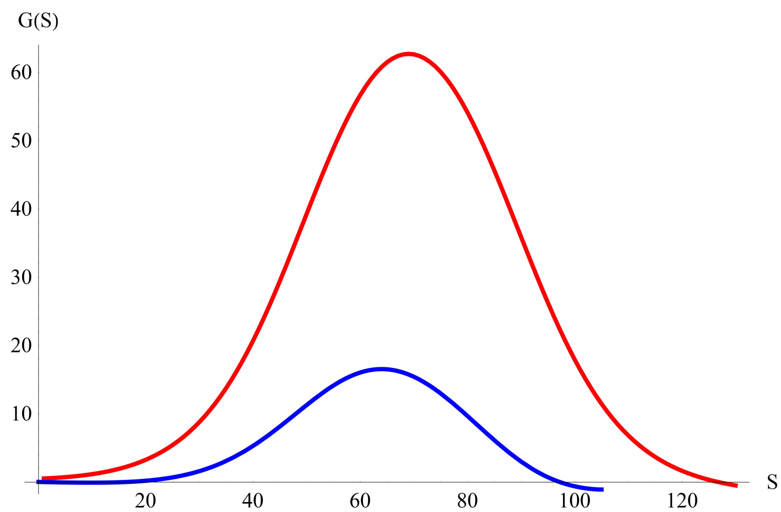


Figure 12 : Benchmark model with multiple equilibria and model with mitigation policy having a unique equilibrium

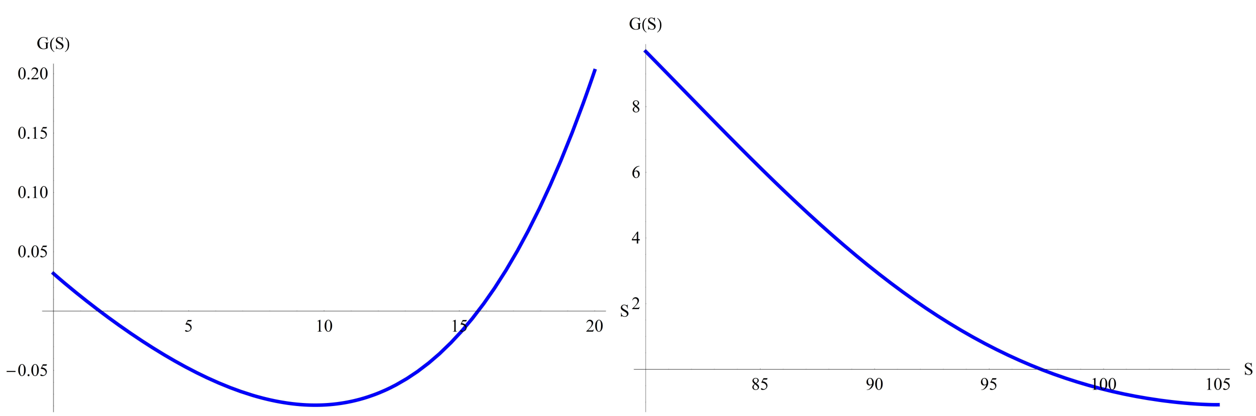


Figure 12.1 : Benchmark model with multiple equilibria

Mitigation policy aims at increasing the environmental quality, which helps to decrease the endogeneous hazard rate. Thus, this policy helps to escape an economy from a possibility to have a multiple equlibria economy that we show in the following figure ;

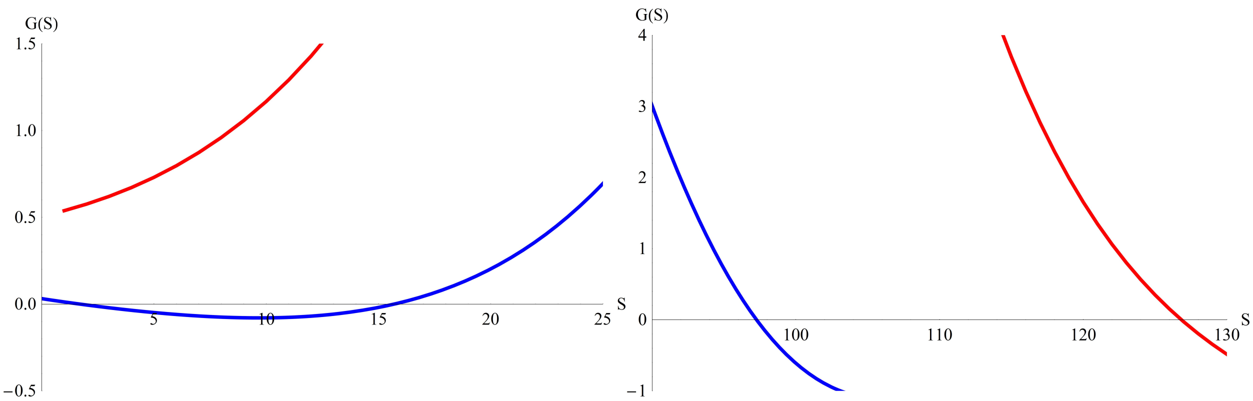


Figure 12.2 : Low and high steady states for benchmark model and model with mitigation policy (Red and blue curves represent benchmark model and model with mitigation policy.)

As it can be easily seen on the Figure 12.2, when there exists mitigation policy, the economy does not trap to low equilibria as $G(S)$ curve (red thick curve) shows the presence of an unique equilibrium. Another remark to be made is that economy has a higher resource stock at steady state with mitigation policy which would increase the long term consumption level of agents.

6.3 Optimal Mix of Adaptation and Mitigation Policy

The optimal policy mix of adaptation and mitigation is crucial as mentionned to avoid environmental inequalities across regions and countries. For this reason, it is crucial to find the optimal trade-off between adaptation and mitigation policy. For this reason, we are also focusing on the design of this optimal policy mix in our numerical analysis. We set $\eta = 5$ and $\bar{h} = 0.5$. Other parameters are the same as presented above.

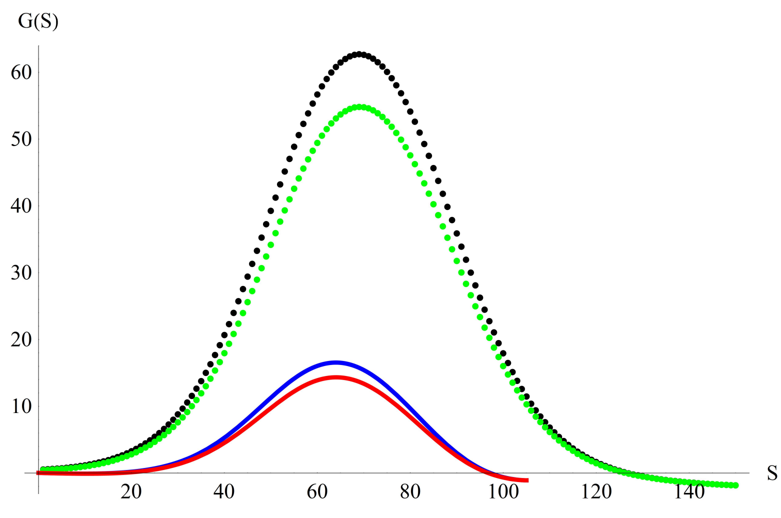


Figure 13 : $G(S)$function (Blue, red, black and green curves represents model without policy, with only adaptation, with only mitigation and with adaptation/mitigation policy respectively)

We observe from the green curve representing $G(S)$ function for optimal policy mix that there exists a unique equilibrium as this function intersects only once the x-axis. Thus, we conclude that it is possible to avoid a multiple equilibria economy with an optimal adaptation/mitigation policy and having a higher consumption and resource stock at the same time. For the sake of clearty of graphics, we split $G(S)$ in two parts in order to see where there are intersection points, as made for only adaptation and only mitigatin policy in Figure 12.2.

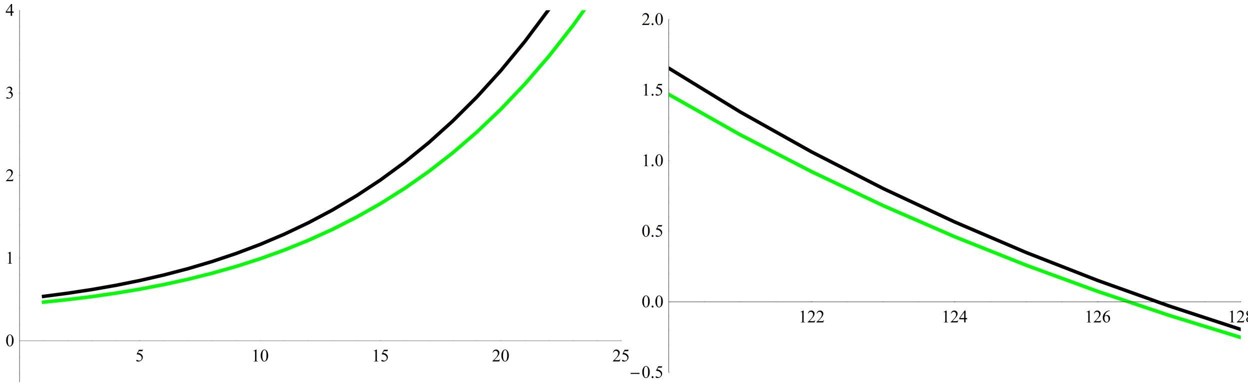
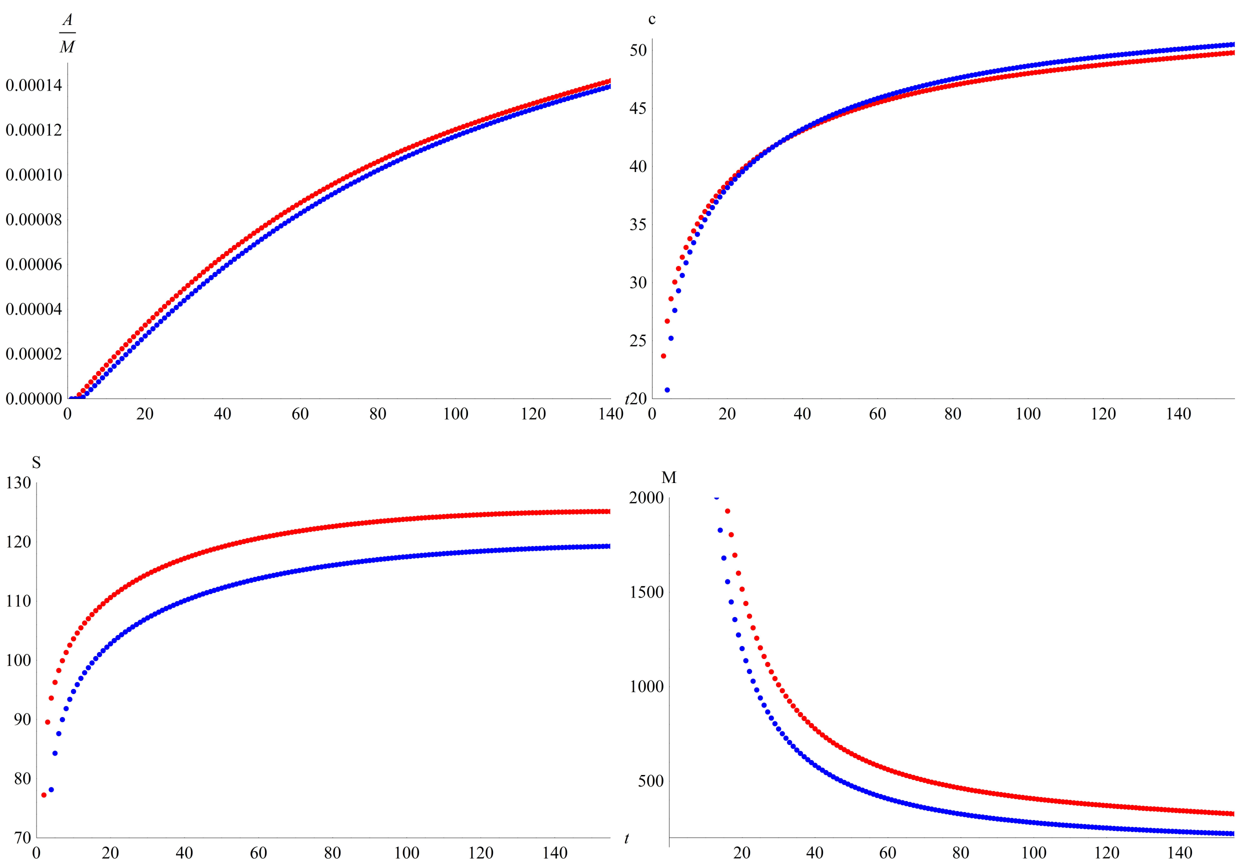


Figure 14 : $G(S)$ function splitted in two parts (Black and Green curves holds for only mitigation and adaptation and mitigation policy respectively)

We see very easily that there exist a unique equilibrium with the policy mix of adaptation and mitigation. We remark that when there exists a policy mix of adaptation and mitigation, resource stock tends to decrease slightly relatively to the model with only mitigation policy, which is an intuitive result as adaptation policy decreases the environmental quality.

Another interesting analysis to have a better insight of the optimal behavior for policy implementation could be to focus on the magnitude of the risk. How a social planner reacts optimaly when she faces a lower/higher risk ?



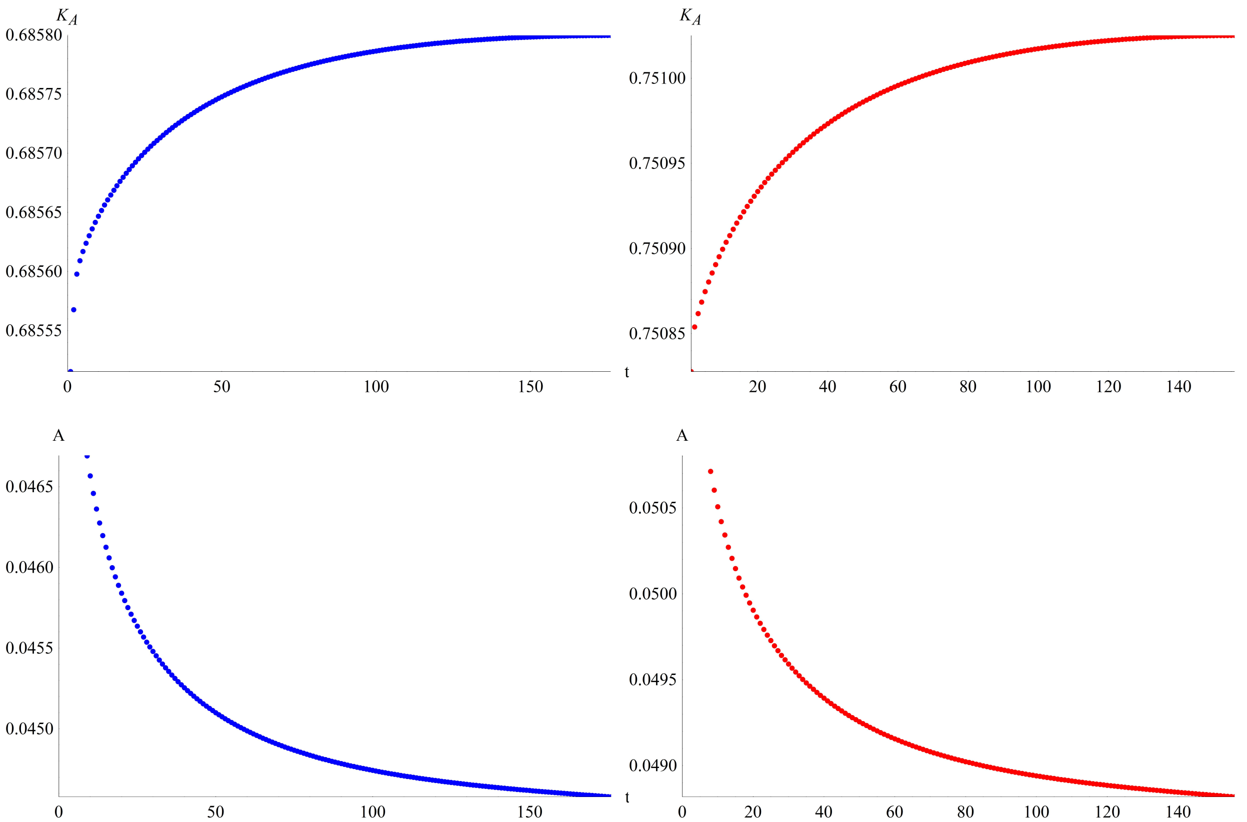


Figure 15 : Optimal Transitional Dynamics (Red curves represent an economy facing a higher risk)

It is interesting to see that social planner gives more weight to adaptation investment than mitigation when she faces a higher risk. This result is somewhat plausible when we remark that social planner would like to decrease the amount of penalty, which is a more probable event when the hazard rate is higher. However, this does not mean that social planner decreases its mitigation level. When there is a higher hazard rate, she increases also the mitigation level but the increase of adaptation investment is higher than mitigation level, which case would cause a multiple equilibria if she faces a very high risk in which situation she will give more weight to adaptation investment.

Thus, as higher risk implies higher adaptation investment, which decreases the resource stock as a result. In this case, economy will again invest more in adaptation capital, which would increase the hazard rate. When this vicious cycle's "more adaptation capital - less resource stock" effect dominates the mitigation activities "decreasing hazard" feature, economy will start to suffer from a multiple equilibria.

We observe that society becomes more precautionnary concerning the exploitation of natural resources an consumption behaviours, which result has been pointed out in Bommier et al. (2015) and Tsur and Zemel (1996) which state that higher risk implies precautionary behaviour in both resource management and consumption.

7 Concluding Remarks

In this article, we studied the optimal policy mix of adaptation and mitigation and discussed how this policy mix should be implemented in a manner that avoids environmental inequalities across different regions and countries. We showed that excessive investment in adaptation capital could cause a multiple equilibria economy. For that reason, policy makers must be precautious about this inequality effect that could adaptation policy cause when she implements the policy mix. Recent reports of IPCC 2014 insists on increasing adaptation capital in order to decrease the vulnerability of developping countries, especially Sub Saharan

African, Latin American and South Asian countries. For the reasons presented within this study concerning inequalities that can arise, we argue that investing more in adaptation capital should not give an incentive for policy makers to invest less in mitigation activity.

One of the main results of this article is that in a world with catastrophic events, which could face a multiple equilibria (i.e environmental inequalities), adaptation capital can trigger this inequalities and adaptation policy, by increasing the hazard rate, could cause catastrophic events. Based on this reasoning, we argue that the adaptation capital advocated in order to challenge the climate change could be the source of the environmental problems. However, this position does not mean that we are rejecting adaptation capital but we are defending the idea that only adaptation policy is not a solution and must be coupled with mitigation activity, in order to avoid environmental inequalities and also to decrease the frequency of catastrophic events, by decreasing the hazard rate.

We show that when a sufficiently high catastrophic event probability, by causing a multiple equilibria, decreases the consumption level and makes the agents trapped at low environmental quality/consumption steady state level less sensitive to environmental quality. This one is due to high marginal utility of consumption. As agents at this equilibrium are impatient and less willing to postpone their consumption, the effect of hazard rate is less important. This does not mean that catastrophic event probability is not important concerning the trade-off between consumption and catastrophic event. Recall that the multiple equilibria phenomena is caused by catastrophic event probability. In this case, we can argue that implementing only adaptation policy will increase the hazard rate not solve the inequality problem.

We also make a numerical illustration by commonly used functional specifications and reasonable calibration that multiple equilibria phenomena (i.e environmental inequalities) due to catastrophic event possibility and to inappropriate policy implementations is not only a theoretical possibility. We show numerically how to implement an optimal policy mix of adaptation and mitigation in way that inequalities disappear.

Numerical illustration shows that a given higher risk of catastrophic event increases both adaptation and mitigation investment but adaptation investment increases much more relatively to mitigation activity. Another striking result is that when a unique equilibrium economy implements only adaptation capital and starts to suffer from multiple equilibria phenomena, the regions trapped to low equilibrium accumulates more adaptation capital than regions at high steady state. This result makes sense because low steady state countries face a higher risk. In this case, we argue that advocating heavily investment in adaptation policy without giving the sufficient importance to mitigation activities could itself increase the hazard rate and the frequence of catastrophic events.

References

A.Ayong Le Kama, A.Pommeret, 2016, Mitigation and adaptation are not enough: turning toemissions reduction abroad.FAERE Working Paper, 2016.01.

Bommier, A., Lanz, B and Zuber, S., 2015, "Models-as-usual for unusual risks? On the value of catastrophic climate change," Journal of Environmental Economics and Management, Elsevier, vol. 74, pages 1-22.

Boyce, J.K, 2007, Is Inequality Bad for the Environment?, Political Economy Research Institute, Working Paper Series, University of Massachusetts Amherst, Number 135.

Boyce, J.K and Torras, M.1998, Income, inequality, and pollution: a reassessment of the environmental Kuznets Curve, Ecological Economics, 1998, vol. 25, issue 2, pages 147-160

Bréchet, T., Hritonenko, N. and Yatsenko, Y., 2013, Adaptation and mitigation in long-term climate policies. Environmental and Resource Economics 55, 217–243

Buob, S., and G. Stephan 2011. To mitigate or to adapt: how to combat with global climatechange, European Journal of Political Economy.27, p.1-16.

Constant K., 2015. "Environmental Policy and Inequality: A Matter of Life and Death," Working Papers halshs-01174052, HAL.

Clarke, H. and W. Reed ,1994, “Consumption/pollution tradeoffs in an environment vul-nerable to pollution-related catastrophic collapse,”Journal of Economic Dynamics andControl, 18, pp. 991 – 1010

Cropper, M. ,1976, “Regulating activities with catastrophic environmental effects,”Journalof Environmental Economics and Management, 3, pp. 1 – 15.

Gjerde, J., S. Grepperud, and S. Kverndokk,1999, “Optimal climate policy under thepossibility of a catastrophe,”Resource and Energy Economics, 21 (3-4), pp. 289 – 317.

IPCC report - Climate Change 2014: Impacts, Adaptation, and Vulnerability

Kane S. and J.F. Shogren 2000. Linking adaptation and mitigation in climate change policy,Climatic Change 45, 75-102.

Kempf, H. and Rossignol S., 2007. "Is Inequality Harmful For The Environment In A Growing Economy?," Economics and Politics, Wiley Blackwell, vol. 19(1), pages 53-71, 03.

Levinson, A and Taylor, S., 2008, Unmasking the Pollution Haven Effect, International Economic Review Volume 49, Issue 1, pages 223-254, February 2008

McAusland, C., 2003. "Voting for pollution policy: the importance of income inequality and openness to trade," Journal of International Economics, Elsevier, vol. 61(2), pages 425-451, December.

Michel P. 1982, On the Transversality Conditions in Infinite Horizon Problems. Econometrica 50, 975-985.

Morgano B. et al. 2014, Primary forest cover loss in Indonesia over 2000–2012, Nature Climate Change 4, 730–735 (2014) doi:10.1038/nclimate2277

Nordhaus, W. D. , 2008, A question of Balance: Weighting the options on global warmingpolicies: Yale University Press.

Parks B. and Roberts J.T, 2008, Inequality and the global climate regime: breaking the north-south impasse, Cambridge Review of International Affairs, Volume 21, Number 4

Ren, B. & Polasky, S., 2014. "The optimal management of renewable resources under the risk of potential regime shift," Journal of Economic Dynamics and Control, Elsevier, vol. 40(C), pages 195-212.

Schumacher, I., 2009, Endogenous discounting via wealth, twin-peaks and the role of technology, Economics Letters 10, 78-80.

Strulik H. 2012, Patience and prosperity. Journal of Economic Theory 147, 336- 352.

Stern, N. ,2007, The Economics of Climate Change: The Stern Review: Cambridge (UK):Cambridge University Press.

Stiglitz, J. 2014, Inequality and Environmental Policy, Resource 2020 Lecture

Tsur, Y. and A. Zemel, 1996, “Accounting for global warming risks: Resource managementunder event uncertainty,”Journal of Economic Dynamics and Control, 20, pp. 1289 –1305.

Tsur, Y. and A. Zemel, 1998, “Pollution control in an uncertain environment,”Journal of Economic Dynamics and Control, 22, pp. 967 – 975.

Tsur, Y. and A. Zemel, 2006, Welfare Measurement under Threats of Environmental Catastrophes, Journal of Environmental Economics and Management, 52:421-429

Tsur, Y. and A. Zemel, 2008, Regulating Environmental Threats, Environmental and Resource Economics, 39:297-310

Tsur, Y. and A. Zemel, 2009, Endogenous discounting and climate policy, Environmental and Resource Economics 44(4):507-520

Tsur, Y. and A. Zemel, 2015, The management of fragile resources: A long term perspective, Environmental and Resource Economics (in press).

Weitzman, M. 2009, “On modeling and interpreting the economics of catastrophic climatechange,”The Review of Economics and Statistics, 91 (1), pp. 1 – 19.

Zivin, J.G., Neidell, M., 2012, The impact of pollution on worker productivity, American Economic Review, Volume 102, pages 3652-3673